A COMPENDIUM OF Algebra.

CONTAINING
Plain and *Easie Rules*, for the *Speedy attaining* to that *ART*.
Exemplified by various *PROBLEMS*, with the *Solution* of their *Æquations* in *Numbers*.

BY

A *New* and *General Method* of *Resolving* all kind of *Æquations*; with great *Ease* and *Expedition*, very *Different* from all others yet Extant.

APPLIED

To *Squaring* the *Circle*, *Making* of *Sines*, *Tangents* and *Logarithms* with great Facility.

ALSO

An *Appendix* concerning *Compound-Interest* and *Annuities*.

The Second *Edition* Corrected.

By *JOHN WARD*,
Teacher of the *Mathematicks*, at the *Globe* in *Fleet street* near *Fetter-lane* end.
Heretofore a *General Gauger* in the *Revenue* of *Excise*.

LONDON,
Printed for *John Taylor*, at the *Ship* in St. *Paul's Church-yard* 1698.

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TO THE

Honour'd, and Learned

Mr. *JOSEPH RAPHSON*,

Fellow of the *Royal Society*.

SIR,

THE *Mathematical Sciences* have always been found of such general Use and Advantage amongst Mankind, That 'tis no wonder all Ages have produc'd so many Great and Eminent Persons, who have employ'd their Studies for the Improvement of those Excellent Arts, wherein their Labours have had great Success; such were *Euclid*, *Archimedes*, *Ptolomy*, *Galilæus*, &c. But to omit all others, and come to our own Country and Age, What great Improvements have been lately made? To instance only that of *Logarithms* (the Noble Invention of the Noble Lord *Napeir*) so useful and necessary in the most considerable Parts of *Mathematical Operations*. Not inferiour to any of these, is your own Discovery of Resolving *Adfected Æquations*, the Value and Advantages of which, succeeding Ages will be enforc'd to acknowledge, as

that which has not only awaken'd, but encourag'd the Sublimest of *Mathematical* Genius to proceed in the further Discoveries of such Mysteries as lie yet conceal'd, and by the more Supine have been thought Impossibilities. Among the rest, I (although the Meanest, yet not the least Laborious) have presum'd to give the World a Taste of the Effects of some Years Experience. And these my first Fruits I humbly Present to your self (worthy Sir) as being a Person, who for your peculiar Learning, Pains and Industry bestowed on this particular Study, may deservedly be deem'd the most Competent and Proper Judge of these ensuing Pages, which if you are pleas'd to Accept and Approve, you will thereby become my Patron to protect me from the Censures of others, I am

SIR,

Your most Obliged,

And Humble Servant,

JOHN WARD.

TO THE
READER.

I Have adventur'd in this Critical Age, to present the World with these few Sheets, which are a Compendium of (that Universal Part of the Mathematics) Algebra; not insensible that several very Eminently Skilful in all Parts of Mathematical Literature, have both Largely and Learnedly handled this most difficult Subject: The Thoughts of which almost prevented my Attempting any thing of this Nature. But when I consider'd that most of those Authors Works were so voluminous, that many Persons who have a Mathematical Genius or Fancy tending that way, might perhaps want Time, or other Conveniences suitable to the Perusal of such Authors; and not only so, but several very well versed in other parts of those Studies, have wholly declined this Art, being deterr'd from it by the Difficulty and Tediousness of Resolving Affected Æquations into Numbers, there never having been in any Book that I have had the opportunity of perusing, (Mr. Raphson's Analysis Æquationum Universalis excepted) any easie Method prescribed for effecting the same; which I was most certain I could perform with great Facility and Expedition.

These Considerations prevail'd with me to believe, That if there were such a Treatise as began with the first Rudiments of Algebra, *and proceeded gradually through the several Parts of it, plainly shewing how to form, and bring any* Problem *to an* Æquation, *and then shew how to Resolve that* Æquation *into Numbers, it would undoubtedly be very Acceptable to all Lovers of Art.*

For these Reasons I composed this Compendium, *contracting the same (both for Convenience of Price and Portability) into as narrow a Compass as possibly I could; (though the Contents of it would have filled a Larger Book,) and yet I presume there is not any thing omitted that might conduce towards the Learners Attaining to a perfect Knowledge of this Mysterious Art, which is indeed the very Principal or Kernel of all the* Mathematical *Sciences, as being the Inventive Part.*

To be brief, the whole is Methodical, Plain and Easie, the Rules being digested and freed from all Obscurity, and explain'd by various Examples in each part of it, beginning with (what I call) Algebraic Arithmetic, *treating of every Part distinctly.*

Next, I introduce the Learner into the use of the same, by Plain and Easie Questions in Numbers, thereby teaching him how to bring Quantities to an Æquation, *&c. Then I shew how to*

Extract

To the Reader.

Extract the Roots *of all simple Powers, how high soever they are* (viz. *of the* Square, Cube, Biquadrate, Sursolide, *&c.*) *by a New and Easie Method, never before Published.*

Then I proceed to the solving *of Geometrical* Problems, *fitted for the raising various Forms of Adfected* Æquations; *and Resolve them into Numbers, laying down the Construction or Rise of the* Theorems *used in their* Solutions.

In short, the Reader will meet with several Compendious and Useful Contractions in this small Treatise, more then needs to be here Numbered; sure I am, it will fully Answer the Title Page; however the rough and unpolished Style in which it is delivered (for I don't pretend to be a Linguist) may perhaps leave room for the Cavils of Critics, especially those that seek for opportunities; for such I know there are, that will find Fault and pick any Quarrel when out-done, or contradicted in their common Practice. Not to such, but to the Ingenious, I willingly submit; hoping this small Tract will meet with their candid and favourable Acceptance; which is the only thing desired and aimed at, By,

Their Real,

And Hearty Well-wisher,

J. W.

Errata.

Page	L.	For these Errors,	Read these Corrections.
1	14	*obtruce*	*abstruce.*
7	15	$a+a$	$a+b$
	20	$8-6=14$	$8-6=2$
11	14	*prefix.*	*Annex.*
	23	$+2b$ — $2ad$	$-2b$. -2
	25	-——	*blot out, both and thus*
12	18	*Quanties:*	*Quantities.*
14	19	$a^3+3bba+bbb$	$a^3-3bba+bbb$
	28	$-4c$.	$+4c$.
15	1	*Quanties.*	*Quantities.*
22	13	$f-c$	$f+c$
	21	$bca-cca$	$bcd+ccd$
25	21	*Compassion.*	*Composition.*
27	25	*be stated.*	*being stated.*
28	23	$+d$	$-d$
31	17	$a-\frac{1}{2}ba=\surd$ &c.	$a-\frac{1}{2}b=\surd$ &c.
38	29	$y=12$	$y=24$
39	4	——	*the* 4th. *Divided by b.*
50	17	$r+e=6473$	$r+e=64783$
51	4	*half the remainder.*	*halve the remainder.*
56	16	$x=\frac{D}{r+2ee}=e$	$\frac{D}{r+2e}=e$
61	15	$rr=450$	$\frac{1}{2}rr=450$
64	8	*prob* 1.	*prop* 1.
67	16	26170, 66 $=\frac{1}{3}$ G.	96170, 66 $=\frac{1}{3}$ G.
69	21	*new* $r.=8,955$	*new* $r=8,055$
79	18	$:c+\frac{3RRa-aaa}{2RR}$	$:\frac{1}{2}c+\frac{3RRa-aaa}{2RR}$
85	15	*calls us.*	*tell us.*
87	26	*from the same*	*for the same.*
107	3, 10	*Account.*	*Amount.*
	21	100 : 6 : : 1 : 1, 06	100 : 106 : : 1 : 1, 06
109	13	1 : 1, 06 : : 100 : 6	1 : 0, 06 : : 100 : 6

A

A COMPENDIUM OF ALGEBRA.

CHAP. I.

Of the Nature of Algebra, *and Definition of the* Symbols, *or* Characters *used therein*, &c.

I Shall not undertake (in this ſmall Tract) to give an Hiſtorical Account of the Original, or Antiquity either of this Art, or its Name; but confine my ſelf within the Limits of this Deffinition, that (that which is now called) *Algebra*, is an Art by (*the help of*) which the moſt difficult and ~~obstruse~~ Queſtions, either in *Arithmetick*, or *Geometry*, may be Reſolved, by an Univerſal Method of Reaſoning, and comparing of known Quantities with thoſe that are unknown.

And this way of arguing, or comparing of Quantities with Quantities, not only diſcovers an Anſwer to the Queſtion propoſed, but at the ſame time doth raiſe (and demonſtrate) Theorems, or Rules, for the Reſolving of all Queſtions in the like Nature.

And from hence it is, that ſo many Curious and Uſeful Theorems, Canons, or Rules are firſt diſcovered, which are of excellent uſe in the Practice, but would for ever lie hid were it not for this Art.

This *Algebraick Arithmetick*, is performed by ſubſtituting of Alphabetical Letters, to repreſent as well the known, as the unknown Quantities concern'd in any Problem, (be it Arithmetical, or Geometrical.) And to theſe Subſtituted Letters, are prefixed certain Symbols, Notes or Characters, by the help of which, the Quantities (in their ſubſtitutes) are ſo connected, and ordered, that thereby they become capable of being compared, and moulded by Addition, Subſtraction, Multiplication, and Diviſion, *&c.* according to the Deſign or Nature of the Queſtion: And this is ſo done, as tho all the Quantities concern'd therein, (as well the ſought as the given) were really known; till at laſt the given Quantities comes to an Æquation (or Equality) with the Quantity that's ſought, (or ſome Power thereof) that ſo it may be found.

In all which various Operations and Compariſons, all the Quantities concern'd in the Queſtion, remain in their Species ſo plain and clear, that they neither burden the Memory, nor rack the Fancy; but on the contrary help both, by repreſenting to the Eye (at once) the Proceſs of each Operation, uſed throughout the whole Management of a Queſtion.

The Method of noting down theſe Subſtitutes in this Tract is thus. I make choice of repreſenting the Quantity ſought (be it a Line, or Number) by the ſmall (*a*,) and if more than one are ſought, I make uſe of the Vowels (*e*,) or (*y*,) &c. to repreſent them. And for the given Quantities,

tities, (be they either Numbers, Lines, or other Magnitudes,) I represent them by the Consonants *b, c, d, f, g,* &c. And for Distinction I note (as usual) all Schemes with the Great or Capital Letters, and work all Operations with the small Letters.

If any Quantity be taken more than once, prefix its Number; as $6a$, $5b$, $9c$, $3\frac{1}{2}d$, &c. these stand for *a*, taken Six times, or Six times *a*, Five times *b*, &c And all Numbers thus prefixed to any Quantity, are called Coefficients (or fellow Factors) but if the Quantity have no prefixed Number, (or Coefficient) then such Quantity stands for itself once taken, and is supposed (or understood) to have an Unit prefixed to it, as *a*, is $1a$, or *b*, is $1b$, *&c.*

The Signs made use of in this Tract, for the more short and quick expressing of Words, are the most General now in use, and are those following.

N. B. Note, That it is very requisite for the Young Learner of this Art, to be very perfect in the knowledge of the Signs, and their Significations, before he proceeds further therein; that is, before he intermedles with any of the following Rules.

Also I advise him to be very ready in one Rule, before he undertakes the next; that is, to be expert in, and understand *Addition*, before he medles with *Substraction*; and that of *Substraction*, before he undertakes that of *Multiplication*, &c. because they have each a Dependency one upon the other.

And by this method of proceeding, he may make himself Master thereof, or at least arrive to a competent knowledge therein, with ease, and in a very short space of time.

Signs.	*Names.*	*Significations.*
$+$	*More.*	The Affirmative Sign, and Sig[n] of Addition, as $a + b$, ſignifiet[h] that the Quantities, a, and b, ar[e] to be added together.
$-$	*Leſs.*	The Negative Sign, and Sig[n] of Subſtraction, as $a - b$, ſignifieth that the Quantity b, is to be taken from that of a.
$\times$	*Into.*	The Sign of Multiplication, as $a \times b$, ſignifieth that a, is to be Multiplied into b, but oftentimes the Sign is omitted, yet is underſtood, as $a\,b$, is $a \times b$.
$\div$	*By.*	The Sign of Diviſion, as $a \div b$ ſignifieth that a, is to be Divided by b; yet this Sign is alſo often omitted, as $\frac{a}{b}$ is $a \div b$, or thus, $b)\,a\,($
$::$	*Proportion.*	The Sign of Disjunct Proportion, as in the *Golden Rule*, and is thus, $a : b :: c : d$. to be read in this manner, as a, is to b, ſo is c, to d.
$\div$	*Continued.*	The Sign of Continued, or Geometrical Proportion; that is, as $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa}$ &c.
$\because$	*Ergo.*	Signifying the Product of the Two Extreams, is equal to that of the Means, and [$\because$] is placed in the Margin after Proportion.

Signs.	*Names.*	*Significations.*
$=$	*Equal.*	The Sign of Æquation, or Equal to, as $a = b$, signifieth that a is equal to b, &c.
@	*Involve.*	The Sign of Involution, and is placed in the Margin of a Process, to signify, that such Quantity, or Æquation to which it refers, is to be Multiplied by it self, *&c.*
ω	*Evolve.*	The Sign of Evolution, and is placed in the Margin, to signify the contrary to Involution, *viz.* The Extraction of the Root, *&c.*
$\surd$	*Root.*	The Sign of Irrationallity, or of a *Surd Root*; over which, if a Number be placed, it signifieth of what degree that Root is: And this Number is called the *Index*, thus

$\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$ Denotes the *Square*, *Cube*, *Biquadrate*, *Sursolide* *Root*, &c.

$\surd$: Signifieth an *Universal Root*, or *Root of Roots.*

$>$	*Greater.*	The Sign of Majority, as $a > b$, signifieth that a is greater than b.
$<$	*Lesser.*	The Sign of Majority reversed, as $b < c$, signifieth, that b, is Lesser than c.
$\square$	*Square.*	Signifieth the Square of any Numbers, or Quantities, or of an Æquation, as $\square\ a + b + c$, &c. is the Square of a, more b, more c.

The chief of these Signs, or Characters wherewith the Substituted Quantities (or Letters) are connected, be More, and Less, *viz.* The Sign $+$, and Sign $-$; where *Note*, That the Sign always belongs to the Quantity that follows it, (or is to the Right Hand thereof,) and not to that which is before it. And if a Quantity hath no Sign prefix'd before it, (as generally the Leading Quantity hath not) it's then understood to have the Sign $+$ prefixed, as a, is $+ a$, or $b + d$, is $+ b + d$, &c.

If several Quantities be connected, or engaged in any Question, they may stand in any order at pleasure; as $a + b - d$, may stand thus, $a - d + b$; or thus, $- d + a + b$, &c. being still the same, tho differently placed; and these are called Compound Quantities, that is, when Two or more different single Quantities are connected together by their Signs, of which more in its due place.

The Method used in this Tract, of placing Figures, and Signs in the Margin, according to the several Steps, or Process used in forming of an Æquation, is the same with that of Mr. *Branckers*, (or rather Dr. *Pells*,) in his *Introduction to Algebra*, Printed 1668, (which hath of late been followed by others) and is thus to be understood.

The Numbers 1, 2, 3, 4, *&c.* in the small Column, do shew the several Steps used in the forming, or bringing the Quantities concerned in any Question to an Æquation: And the Numbers placed in the Margin, have relation to those in the small Column, and the Signs placed betwixt the said Numbers in the Margin, do shew, how the Quantities that stand against the like Numbers in the small Column have been ordered, either by Addition, Substraction, or otherwise. And this Method of Registering the several Processes, is very useful, especially in long and tedious Operations, as will more fully appear in the Practice thereof. An Example or Two will help to explain the same better than Words.

Suppose

Suppose it were required to set down the Sum of the Two Quantities, a, and b, according to this Method.

Thus

	1	a
	2	b
1 + 2	3	$a + b$

Having placed down the Quantities a and b, against the Figures 1, 2, in the small Column, also against 3, must be placed 1 + 2 in the Margin, which denotes that the Quantities in the First and Second Steps, are added together, and those in the Third Step, are their Sum.

To illustrate this in Numbers, suppose $a = 8$, and $b = 6$.

		Quantities.	Numbers.
	1	a	8
	2	b	6
1 + 2	3	$a + b$	$8 + 6 = 14$

Again, suppose it were required to set down the Difference of the same Two Quantities.

Thus

	1	a	8
	2	b	6
1 — 2	3	$a - b$	$8 - 6 = 2$

Again to set down their Product.

Thus

	1	a	8
	2	b	6
1 × 2	3	ab	$8 \times 6 = 48$

These Definitions of the Signs, (or Notes) Method, *&c.* being premised, and pretty well understood by the young Learner, (for so they ought to be,) He may then proceed to the several parts of *Algebraick Arithmetick*, viz. *Addition*, *Substraction*, *Multiplication*, and *Division*, &c.

In the delivery of which, I shall begin with single whole Quantities, in each distinct part thereof, then proceed in the like Order with Compound Quantities, and so on to Fractions, treating of each separately, with what plainess and brevity I can.

CHAP.

CHAP. II.

Of the several parts of Algebraick Arithmetick, *and how the same are performed.*

§. 1. Addition *of Single whole Quantities.*

Rule 1. IF the Quantities be a like, and have like Signs, add the Coefficients (or prefixed Numbers) and adjoyn the Quantities with the same Sign.

Thus	1	a	$-3b$	$+5bc$	$-6abc$
	2	a	$-5b$	$+4bc$	$-7abc$
$1+2$	3	$2a$	$-8b$	$9bc$	$-13abc$

Rule 2. When the Quantities be alike, but have unlike Signs, substract the Coefficient from each other, and adjoyn the Quantities with the Sign of the Greater.

Thus	1	$3a$	$-5b$	$-9bd$	$10abcd$
	2	$-a$	$+7b$	$+7bd$	$-12abcd$
$1+2$	3	$2a$	$+2b$	$-2bd$	$-2abcd$

Rule 3. But when the Quantities be unlike, (be the Signs alike or unlike) set down the Quantities with their respective Coefficients, without altering their Signs, and thence ariseth Compound Quantities.

Thus	1	a	a	$-7bcd$
	2	b	$-b$	$+5bdg$
$1+2$	3	$a+b$	$a-b$	$5bdg-7bcd$

Also thus	1	a	$-5a$
	2	b	$+7bc$
	3	c	-40
$1+2+3$	4	$a+b+c$	$7bc-5a-40$

§. 2. Substraction *of Single whole Quantities.*

General Rule.

CHange, (or at least in your Mind suppose to be changed) all the Signs of the Subtrahend; then add, or collect all the Quantities together, (as before in Addition) and that Sum will be the Remainder, or Difference sought.

For Example, Suppose it were required to find the Difference of these Quantities.

$2a$	$8b$	$-7bc$	$2ba$
a	$5b$	$-4bc$	$2ba$

According to the Rule, the same Quantities will stand, when their Signs are changed thus.

	1	$2a$	$8b$	$-7bc$	$2ba$
	2	$-a$	$-5b$	$+4bc$	$-2ba$
$1-2$	3	a	$3b$	$-3bc$	0

But this may be done by supposing the Signs changed, without setting them down again.

Thus	1	$5a$	$-4bc$	$-5bd$	0
	2	$-2a$	$+5bc$	$-8bd$	$-2ba$
$1-2$	3	$7a$	$-9bc$	$+3bd$	$+2ba$

Again,	1	a	$-4b$	$+5bcd$
	2	b	$-3a$	$-7baf$
$1-2$	3	$a-b$	$+3a-4b$	$5bcd+7baf$

Thus Compound Quantities, are produced by *Substraction*, as they were by *Addition.*

Note, That *Substraction* is proved, by adding the Subtrahend to the remainder; as in vulgar Arithmetick.

Thus	1	$7b$	$-5bc$	$-4abd$
	2	$-2b$	$+3bc$	$-3cfg$
$1-2$	3	$9b$	$-8bc$	$+3cfg-4abd$
$2+3$	4	$7b$	$-5bc$	$-4abd$

§. 3. Mul-

§. 3. Multiplication *of single whole Quantities.*

Rule 1. IF the Quantities have no Coefficients, and have like Signs, joyn them together, whether they be like or unlike, and to the Product prefix the Sign +.

Thus	1	a	ab	$-bc$	$-ab$
	2	b	cd	$-df$	$-d$
1 × 2	3	ab	$abcd$	$+bcdf$	abd

Rule 2. If there be Coefficients, Multiply them, and to their Product joyn the Quantities, (set together as before.)

Thus	1	$5a$	$7bc$	$-24bc$
	2	b	$4ad$	$-6df$
1 ×2	3	$5ab$	$28adbc$	$+144bcdf$

Rule 3. And when the Quantities have unlike Signs, the Operations are the same with the foregoing; only to the Product prefix the Sign —.

Thus	1	a	$-bc$	$-7b$	$-ba$
	2	$-b$	$+cd$	$+6ad$	$+12$
1 × 2	3	$-ab$	$-bccd$	$-42bad$	$-12ba$

From hence it appears, that like Signs produce the Affermative Sign +, but unlike produce the Negative Sign —.

The Reason thereof may be thus stated: Suppose $a =$ any Affermative Quantity, and let $-b = -2$, then is b equal to 2 less than nothing; (for so are all Negative Quantities.) Now to Multiply any Number or Quantity, implies a putting together of the same, so often as is the Number in the Multiplier, as $a \times 2 = 2a$, or a, twice put together. But if the Multiplier be -2, then it implies the taking away of a, twice; and therefore the Product must needs be $-2a$, that is, $a \times -2 = -2a$, or $a \times -b = -ab$, as above. But to Multiply $-a$, into -2, by the same Reason, it will take away the Defect of a, twice, which is the same as to supply it; therefore $-a \times -2 = +2a$, or $-a \times -b = ab$.

§. 4. Di-

§. 4. Division *of single whole Quantities.*

Rule 1. IF they have like Signs, and no Coefficients, and if there be like Quantities in the Dividend, as there is in the Divisor, cast off such Quantities in both, and set down the other Quantities, with the Sign +.

Thus	1	ab	$abcd$	$-bcf$	$-abc$
	2	a	bc	$-bf$	$-abc$
1÷2	3	b	ad	$+c$	$+1$

N. B. Note, That when the Quantities in the Dividend and Divisor be all the same, then the Quotient is an Unity; as in the last above.

Rule 2. But if the Quantities have Coefficients, divide the Numbers (as in common Arithmetick,) and to the Quotient prefix the Quantites.

Thus	1	$6ab$	$-12abc$	$18cd$	$-8adf$
	2	$3a$	$-3ab$	$3c$	$-2adf$
1÷2	3	$2b$	$+4c$	$6d$	$+4$

Rule 3. When the Quantities have unlike Signs, the Operations are the same; only set down the Quotient with the Sign —.

Thus	1	$-ab$	$+6ab$	$+12cd$	$-8ad$
	2	$+a$	$-3a$	$-4c$	$+4ad$
1÷2	3	$-b$	$+2b$	$-3c$	$-2ad$

But when the Quantities in the Divisor, are not found in the Dividend, set down both ~~both~~ Fraction wise thus,

Thus	1	a	$6bc$	bdf	abc
	2	b	$3d$	abf	$4ac$
1÷2	3	$\frac{a}{b}$	$\frac{2bc}{d}$	$\frac{d}{a}$	$\frac{b}{4}$

§. 5. In-

§ 5. Involution *of single whole Quantities.*

INvolution is performed as *Multiplication* is,(and indeed is no more but *Multiplication* differing only in this, *Multiplication* may be perform'd by different Multiplicators, but *Involution* still retains the same.) If any Quantity (or Quantities) be involved, or drawn into itself, and afterwards into that Product, or again, a third time, into the last Product, *&c.* as manifold as is the power, so great must the Number be that is used to express it; and this Number is called the Index of the power, and is set after the Sign in the Margin.

Thus	1	*a*	— 2 *a*	*bc*
1 ☉ 2	2	*aa*	+ 4 *aa*	*bbcc*
1 ☉ 3	3	*aaa*	— 8 *aaa*	*bbbccc*
1 ☉ 4	4	*aaaa*	+ 16 *aaaa*	*bbbbcccc*
1 ☉ 5	5	*aaaaa*	— 32 *aaaaa*	*bbbbbccccc*

Which may be shortened, by placing of the Indices over the Quantity (or Quanties) according to the height of the Power; by such Method the Work will stand thus.

	1	a	$-a$	bc	*Root*
1 ☉ 2	2	a^2	$+a^2$	$b^2\ c^2$	*Square*
1 ☉ 3	3	a^3	$-a^3$	$b^3\ c^3$	*Cube*
1 ☉ 4	4	a^4	$+a^4$	$b^4\ c^4$	*Biquadratick*
1 ☉ 5	5	a^5	$-a^5$	$b^5\ c^5$	*Surfolide, &c.*

§. 6. Evo-

§. 6. Evolution, (*or Extracting of Roots*) *of single whole Quantities.*

Rule. IF the Quantity have a Root, (which is easily discover'd, by comparing its Index with the Root required,) divide the Index of its Power, by 2, if the Square Root be desired; or by 3, if you require the Cube Root, *&c.* and the Quotient will give the Root required, dealing with the Coefficients, or prefixed Numbers, as in Vulgar Arithmetick.

Thus	1	a^2	$36\,bb$	$81bbcc$
1 ω 2	2	a	$6\,b$	$9bc$

	1	a^6	$8bbb$	$bbbddd$
1 ω 2	2	a^3	$\sqrt{8bbb}$	$\sqrt{bbbddd}$
1 ω 3	2	a^2	$2b$	bd

But if either out of the Indices, or out of the Coefficients, no exact Root can be extracted, according to the Root required; then prefix before it the Sign of the required Root; and thence will arise a *Surd Quantity,* as in the Square Root of the Two last Examples above; and such are some of these following.

	1	a^6	$-\quad 32a^5$	ab	$216d^3$
1 ω 2	2	a^3	$+\sqrt[2]{32a^5}$	$\sqrt[2]{ab}$	$\sqrt[2]{216a^3}$
1 ω 3	3	a^2	$-\sqrt[3]{32a^5}$	$\sqrt[3]{ab}$	$6d$
1 ω 4	4	$\sqrt[4]{a^6}$	$+\sqrt[4]{32a^5}$	$\sqrt[4]{ab}$	$\sqrt[4]{216d^3}$
1 ω 5	5	$\sqrt[5]{a^6}$	$-\quad 2a$	$\sqrt[5]{ab}$	$\sqrt[5]{216d^3}$

Thus I have gon through the Six parts of *Algebraick Arithmetick,* in single whole Quantities, which being well considered, (and rightly understood) the following will be very easie; *viz.* Compound Quantities.

CHAP. III.

The parts of Algebraick Arithmetick *in Compound Quantities.*

§. 1. ADdition of Compound Quantities is perform'd in the same manner, and by the same Rules as single Quantities are; *viz.* by putting together like Quantities, respect being had to the Signs; and in case the Quanties be not alike, connect them together by their Signs, as in the last case of Single Quantities.

Thus	1	$a + e$	$a + e$	$+ 3a - b + 3c$
	2	$a + e$	$a - e$	$- 3a + b - 2c$
$1 + 2$	3	$2a + 2e$	$2a$	c

Again,	1	$a - b + c - 7$	$- 4bc + bcd$
	2	$a - b - c + 3$	$+ 2bc + 3bcd$
	3	$- a + 2b + d + 6$	$+ 4bc - 2bcd$
$1 + 2 + 2$	4	$a + d + 2$	$2bc + 2bcd$

Again,	1	$aa + 2ab + bb$	$a^3 + 3bba + bbb$
	2	$- 4ab$	$- 6bba$
$1 + 2$	3	$aa - 2ab + bb$	$a^3 + 3bba + bbb$

§. 2. *Substraction* in Compound Quantities, is in all respects like that of Single Quantities, care being taken of their Signs as before.

Thus	1	$a + b$	$2a + b + 2c$	$4a + 3b - 75$
	2	$a - b$	$a + b + c$	$- a + 4b + 35$
$1 - 2$	3	$2b$	$a + c$	$5a - b - 110$

Again,	1	$aa + 2ab + bb$	c
	2	$+ 4ab$	$3a - b - 3c$
$1 - 2$	3	$aa - 2ab + bb$	$- 3a + b - 4c$

§. 3. Mul-

§ 3. *Multiplication* of Compound Quanties is the ſame with that of Single Quantities; for the Product of each Member of the Multiplier, into all the Members of the Multiplicand, (reſpect had to the Signs) is the Product.

Thus	1	$3a + 5d$	$2b + c - 4d + f$
	2	b	$2bc$
1×2	3	$3ab + 5db$	$4bbc + 2bcc - 8cbd + 2bcf$

Again at large,

	1	$a - b + c$	
	2	$b - d$	
$1 \times b$	3	$ab - bb + bc$	Product by b
$1 \times d$	4	$- ad + bd - dc$	Product by $- d$
1×2	5	$ab - bb + bc - ad + bd - dc.$	

Again,

	1	$a + d + f - 3$	
	2	$b - c$	
		$ab + db + fb - 3b$	Product by $+ b$
		$- ac - dc - fc + 3c$,	Product by $- c$
1×2	3	$ab + db + fb - 3b - ac - dc - fc + 3c$	

§. 4. *Diviſion* of Compound Quantities, is perform'd like that of Single; and if any Doubts ſhould ariſe therein, they are eaſily reconciled, by conſidering, That *Diviſion* diſſolveth what *Multiplication* puts together; and therefore to be perform'd by contrary Operations.

Thus	1	$3ab + 5db$	$2bcf + 4bbc + 2bcc - 8bcd$
	2	b	$2bc$
$1 \div 2$	3	$3a + 5d$	$f + 2b + c - 4d$

Thus the Product of the Second Example in *Multiplication*, divided by $2bc$, gives $2b + c - 4d + f$.

Examples of Division *at large.*

Suppose the Product of the last Example in *Multiplication*, were given to be divided by $b - c$, the Operation will be thus perform'd.

	1	$ab - ac + db - dc + bf - fc - 3b + 3c$	
	2	$b - c$	
$2 \times a$	3	$ab - ac$	$(+ a$
$1 - 3$	4	$+ db - dc$	$(+ d$
$2 \times d$	5	$+ db - dc$	
$1 - 5$	6	$+ bf - fc$	$(+ f$
$2 \times f$	7	$+ bf - fc$	
$1 - 7$	8	$- 3b + 3c$	$(- 3$
$2 \times \overline{3}$	9	$- 3b + 3c$	
$1 - 9$	10		0
$1 \div 2$	11	$a + d + f - 3.$	Each Quotient Collected.

Oftentimes the whole Process is omitted, only set down with the Sign $\div$ Thus $aa - bb \div a$, or $b + a$) $aa - bb$ (

When the Dividend will not be just wrought off without a Remainder, in such Cases, set all down (like the terms of a Vulgar Fraction,) as in the last Example of Single Quantities.

Thus			
	1	$ab + cb$	$aa + 2ab + cc$
	2	$d + f$	$a + 2b$
$1 \div 2$	3	$\dfrac{ab + cb}{d + f}$	$a + \dfrac{cc}{a + 2b}$

§. 5. *Involution* of Compound whole Quantities, is performed in all respects like *Multiplication*.

Example. Suppose the Square, Cube, or any other Power were required, from the Compound Root $a + b$, or $a - b$, *viz.* a Binomial or Residual Root.

		Binomial.	Residual.
	1	$a + b$	$a - b$
		$a + b$	$a - b$
1 × a	2	$aa + ab$	$aa - ab$
1 × b	3	$+ ab + bb$	$- ab + bb$
1 Ʒ 2	4	$aa + 2ab + bb$	$aa - 2ab + bb$
		$a + b$	$a - b$
4 × a	5	$a^3 + 2aab + abb$	$a^3 - 2aab + abb$
4 × b	6	$+ aab + 2abb + b^3$	$- aab + a2bb - b^3$
1 Ʒ 3	7	$a^3 + 3aab + 3abb + b^3$	$a^3 - 3aab + 3abb - b^3$

Thus are the Square and Cube produced from their Roots; and by this Method of Multiplying by the Root, may any Power be raised from any Compound Quantities given: Other Methods there are, but this I take to be the plainest, (and most Natural) for a Learner's Practice.

Again, Suppose the Square of $a + b + c$, was required.

		Trinomial Root.
	1	$a + b + c$
		$a + b + c$
1 × a	2	$aa + ab + ac$
1 × b	3	$+ ab + bb + bc$
1 × c	4	$+ ac + bc + cc$
1 Ʒ 2	5	$aa + 2ab + bb + 2ac + 2bc + cc$

The Use, and easier Construction of Powers arising from 2, 3, 4, or more Members in the Root, shall be shewed hereafter.

§. 6. *Evolution* of Compound Quantities, is the Converse of *Involution*, (and consequently perform'd by contrary Operations.) Whoever intends to be very dexterous at *Evolution*, or extracting of Roots out of Compound Quantities, remaining in their Species, must be very ready, and understand well the Genisis, or raising of Powers from their Roots; otherwise it will be difficult to discover whether the Given Quantities have a true Root or not.

The Problems in this Tract require no such Operations; yet that it may not appear deficient, observe this easie Rule.

Take so many distinct Quantities, as there are concerned in the Given Power to be evolved, and involve those Quantities (so taken) to the height of the Given Power, (without respect to the Signs;) this new raised Power being compared with the Given Power, will shew whether it have a true Root or not; which if it have, then it's easily evolved: If not, then it is a Surd Compound, and must have its Sign preffixed to it, and is no otherwise to be expressed; till it come to be evolved in Numbers.

But because such Operations are tedious, I shall omit giving Examples thereof in Species, and according to the Design of this Tract, (which is brevity,) proceed to Fractions.

CHAP.

CHAP. IV.

Of Fractions, *or Broken Quantities.*

§. 1. *What is done about vulgar* Fractions *in Numbers, the same may also be performed in Quantities.*

Rule 1. TO bring whole Quantities to a Fraction of a Given Denominator, multiply both together for a Numerator, and place the Given Quantity for the Denominator; thus, $a = \frac{ad}{d}$, and $b = \frac{bcd}{cd}$, also $a = \frac{ac+da}{c+d}$, $d + \frac{dd}{b} = \frac{db+dd}{b}$ &c.

Rule 2. When whole Quantities are to be expressed Fraction-wise, give them an Unity for a Denominator, as $aa = \frac{aa}{1}$ and $ad + bc = \frac{ad+bc}{1}$.

§ 2. *To reduce* Fractions *to a lower Denomination.*

Rule 1. WHen the same Quantities are found both in the Numerator and Denominator, cast them away in both, and a Fraction remains of the same value, as $\frac{abc}{dbc} = \frac{a}{d}$, and $\frac{bcdf}{bdf} = c$. $x + \frac{adf}{df} = x + a$,

Thus the greatest common Divisor brings the Fraction to the lowest Denomination.

Rule.

Rule 2. But when the Fraction is a Compound, the Method to obtain that common Diviſor, is by dividing the Denominator by the Numerator, and the Numerator by the Reſidue, and ſo on (as in Vulgar Arithmetick,) 'till there remain nothing; by the laſt Diviſor, divide both the Numerator and Denominator of the Fraction, and it will be reduced to the Loweſt Denomination.

Example. Let the Fraction propoſed be $\frac{ab + cb}{aa + 2ac + cc}$

	1	$aa + 2ac + cc$, the Denominator,
	2	$ab + bc$, the Numerator,
$1 \div 2$	3	$\frac{a + c}{b}$
$3 \times b$	4	$a + c$ { *Note*, Any Fraction is Multiplied by ſo much as is caſt away out of the Denominator.
		Hence it appears $a + c$ is the greateſt Diviſor,
$2 \div 4$	5	$\frac{b}{a + c}$ the new Fraction $= \frac{ab + cb}{aa + 2ac + cc}$
$1 \div 4$	6	

But if this way be thought too tedious, then diſcover all the Aliquate parts in each, and the Denominator and Numerator being divided thereby, are reduced.

But if after all means uſed, there cannot be found one common Diviſor to both, then are the Fractions Prime to each other.

§. 3. *To bring different* Fractions *into one Denomination, retaining the ſame value.*

Rule 1. This is perform'd (as in *Vulgar Fractions*) by Multiplying the Denominators into each other for a new Denominator, and the Numerators croſs-wiſe into each others Denominator, for new Numerators.

Let

Let the Fractions be, $\frac{a}{b}$, $\frac{c}{d}$, $\frac{f}{h}$, &c.

Reduced will stand thus, $\frac{adh}{bdh}$, $\frac{bch}{bdh}$, $\frac{bdf}{bdh}$, and retain the same value with the first.

By these means Fractions are made fit for *Addition* and *Substraction*, which Two Rules cannot well be perform'd, until different Fractions are reduced to one Denomination.

§. 4. Addition *of* Fractions.

Rule. WHen the Fractions are brought to one Denomination, (as before) add, or substract their Numerators.

Example,

Thus,	1	$\frac{3b}{d}$	$\frac{3c+2b}{d+f}$	$\frac{a-b+de}{c+f}$
	2	$\frac{a}{d}$	$\frac{x+b}{d+f}$	$\frac{a+b-de}{c+f}$
1 + 2	3	$\frac{a+3b}{d}$	$\frac{3c+2b+x+b}{d+f}$	$\frac{2a}{c+f}$

Substraction.

Thus,	1	$\frac{3b}{d}$	$\frac{3c+2b}{d+f}$	$\frac{a-b+de}{c+f}$
	2	$\frac{a}{d}$	$\frac{x-b}{d+f}$	$\frac{a+b-de}{c+f}$
1 — 2	3	$\frac{3b-a}{d}$	$\frac{3c+2b-x+b}{d+b}$	$\frac{2de-2b}{c+f}$

§. 5. Mul-

§. 5. Multiplication *of* Fractions.

Rule 1. MUltiply the Numerators and Denominators respectively into each other, (as in Vulgar Fractions) their Products are the new Fractions.

Note, But first prepare them for the work, by making mix'd Quantities improper Fractions, having first brought down the Fractions to their lowest terms; also make whole Quantities like Fractions, by placing an Unity under them: *As in the first Section of this Chapter.*

	1	$\frac{2a}{b}$	$\frac{2a+b}{d-e}$	$\frac{a-7+b}{d}$
	2	$\frac{3c}{d}$	$\frac{de}{d+e}$	$\frac{a+8}{f-g}$
1 × 2	3	$\frac{6ac}{db}$	$\frac{2ade+dbe}{dd-ee}$	$\frac{aa+a+ba+8b-56}{df+dg}$

Whole Quantities by Fractions stand as followeth.

Thus,	1	$\frac{b+c}{1}$	$\frac{ab-ad}{1}$	$\frac{ac+7}{1}$
	2	$\frac{cd}{f}$	$\frac{b}{c}$	$\frac{a+c}{b+c}$
1 × 2	3	$\frac{bcd-ccd}{f}$	$\frac{abb-abd}{c}$	$\frac{aac+7a+acc+7c}{b+c}$

§. 6. Division *of* Fractions.

Rule 1. D*Ivision* is thus perform'd: Multiply the Numerator of the Dividend, by the Denominator of the dividing Fraction, for a new Numerator, and the other Numerator and Denominator, for a new Denominator.

Suppose

Suppose $\frac{a+b}{d}$ were to be divided by $\frac{c-b}{f}$,

it will stand thus $\frac{a+b}{d} \div \frac{c-b}{f} = \frac{af+bf}{cd-bd}$,

Again, Admit $aa+bc$, were to be divided by $\frac{cb-ad}{c+g}$

it will be $\frac{aa+bc}{1} \div \frac{cb-ad}{c+g} = \frac{aac+bcc+aag+bcg}{cb-ad}$.

Rule 2. When the Fractions are of one Denomination, divide the Numerators, and cast off the Denominator; as $\frac{aacc}{b} \div \frac{ac}{b} = ac$, for $\frac{aaccb}{acb} = ac$.

Again let $\frac{abb}{d} \div \frac{abc}{c} = \frac{b}{d}$ for $\frac{abbc}{abcd} = \frac{b}{d}$.

§. 7. Involution *of* Fractions, *is performed in all respects like* Multiplication.

Thus,	1	$\frac{3a}{b}$	$\frac{4bd}{c}$	$\frac{a+b}{g+x}$
1 ☉ 2	2	$\frac{9aa}{bb}$	$\frac{16bbdd}{cc}$	$\frac{aa+2ab+bb}{gg+2gx+xx}$
1 ☉ 3	3	$\frac{27aaa}{bbb}$	$\frac{64bbbddd}{ccc}$	$\frac{a^3+3baa+3bba+b^3}{g^3+3ggx-3gxx+x^3}$

N. B. If after any of these Operations in the foregoing parts of *Addition*, *Substraction*, &c. there be like Quantities in the Numerator and Denominator, care is to be taken that such Fractions be reduced lower, by some of the former Rules, if possible, before they be ingaged in other Operations.

I shall pass over *Evolution*, because it rarely falls out, that both the Numerator and Denominator have a true Root; therefore if the Sign of the Root be prefixed, it's sufficient, until such Fractions are evolved in Numbers.

CHAP.

CHAP. V.

Of Proportions.

§ 1. PRoportion is either Disjunct, or Continued; Disjunct is, when of Four Quantities, the First is to the Second, as the Third is to the Fourth; if Quantities be thus qualified, then the Product of the two Extreams, is equal to the product of the two Means. And if the product of the Extreams be equal to the product of the Means, then are those Quantities proportional, (16 *Eucl.* 6.) let the Four Quantities proposed be $a, b, c, d,$ if these be proportional, they shall also be proportional, being *Alterned*, *Inverted*, *Compounded*, *Divided*, *Converted*, and in *Mixture*.

Oughtred's Clavis, Chap. 6.

	1	$a : b :: c : d$,	Direct.
1, ∵	2	$ad = bc$	
	3	$a : c :: b : d$,	Alterned.
3, ∵	4	$ad = bc$	
	5	$b : a :: d : c$,	Inverted.
5, ∵	6	$bc = ad$	
	7	$a + b : b :: c + d : d$,	Compounded.
7, ∵	8	$ad + bd = bc + bd$,	that is, $ad = bc$.
Or	9	$a + c : c :: b + d : d$.	
9, ∵	10	$ad + cd = bc + cd$,	that is $ad = bc$.
	11	$a - b : b :: c - d : d$,	Divided.
11, ∵	12	$ad - bd = bc - bd$,	that is, $ad = bc$.
Or	13	$a - c : c :: b - d : d$.	
13, ∵	14	$ad - cd = cb - cd$,	that is, $ad = bc$.
	15	$a : b \pm a :: c : d \pm c$,	Converted.
15, ∵	16	$ad \pm ac = bc \pm ac$,	that is, $ad = bc$.
	17	$a + b : a - b :: c + d : c - d$,	in Mixture.
17, ∵	18	$ac - ad + bc - bd = ac + ad - bc - bd$.	
18, ±	19	$2bc = 2ad$,	that is, $ad = bc$.

From

From the foregoing Operations, it is evident, that Four proportional Quantities may be varied, and intermingled one with the other, and yet retain their true Proportions; which variation may be of good uſe upon ſeveral occaſions.

N. B. And from hence may be learnt, how to convert Analogies into Æquations, or the contrary of Converting Æquations into Analogies.

§. 2. Continued Proportion (or Geometrical Progreſſion) is, when of Three, Four, Five, or more Quantities, the Firſt is to the Second, as that Second is to the Third; and the Second is to the Third, as that Third is to the Fourth, &c. that is, when all the Middle Terms between the Firſt and Laſt Term are both Conſequents and Antecedents; ſuch are in $\div\!\!\div$.

as, $a : b :: b : \frac{bb}{a} :: \frac{bb}{a} : \frac{bbb}{aa} :: \frac{bbb}{aa} : \frac{bbbb}{aaa}$, &c. theſe are in continued Proportion.

Much may be ſaid of the Nature of *Proportions*, for indeed the whole Body of the *Mathematicks*, is nothing but a Compleate Compaſſion thereof.

§. 3. Before we leave this, take one Conſideration more, that if never ſo many Quantities be Proportional, it will be, as one of the Antecedents is to its Conſequent, ſo is the Sum of the Antecedents, to the Sum of the Conſequents. As, $a : b :: c : d :: f : g :: h : k$, &c. Then will $a : b :: a + c + f + h : b + d + g + k$, &c.

CHAP. VI.

Of Æquations, and Questions resolved thereby.

§. 1. QUantities are said to be brought to an Æquation, when some one of those that be sought, (or some Powers thereof, as Squares, Cubes, &c.) is found equal to those that be given. And this is to be obtained by a mutual comparing of equal Quantities, (or things) with equal, by such Methods or Rules, as the Nature of the Problem (or Question) requireth.

Therefore, when any Question is proposed to be resolved, it is highly requisite that the Design or Meaning thereof, be fully and clearly comprehended, that so the Question may be truly stated, or fitted for Operation.

And this part of the work is something difficult; yet having a competent knowledge in the Principles of *Geometry* and *Arithmetick* (together with a good and desiring genious,) a little Practice will render it facile and pleasant.

Having got a clear understanding of the Question proposed, for each Quantity concerned therein, be it a Line, or Number, put or substitute a Letter, as before taught at the beginning of this Tract, (*Page* 2, and 3;) but if Two, or more Quantities in the Question are granted to be equal, they may be expressed by the same Letter.

Having placed down all the concerned Quantities in their due order, (according to the import of the Question) then, consider whether the same be limitted or not; for the discovery thereof, observe the following Rules.

Rule

Rule 1. Whensoever the Number of required Quantities exceeds the Number of the given Æquations, the Question is capable of innumerable Answers.

For Example, Suppose a Question were thus proposed. There be Three Numbers, the First is equal to the Square of the Second, and the Sum of the First and Second, is equal to the Third, What are they Three Numbers. To signifie these Three Numbers, put Three Letters; call the First a, the Second e, and Third y; then $a = ee$, and $a + e = y$, according to the Question. Here the sought Quantities are Three, to wit, a, e, and y, but the given Æquations are but Two; therefore this Question is unlimited. Because for any one of these Three Quantities, any Number may be taken at pleasure. Therefore, unlimitted.

Rule 2. But when the given Æquations, (not depending one upon the other, are as many as the required Quantities, then hath the Question a determinable number of Answers; for then each Quantity concern'd, hath but one single value.

And if the given Æquations be more than the sought Quantities, they not only limit the Question, but oftentimes render it unresolvable, by being inconsistent one with another.

The Question be stated, and found limitted to one Answer, (or at least so bounded, as to have a certain determinable number of Answers,) let all the Quantities be so Ordered, Moulded or Compared, by Adding, Subducting, Multiplying, or dividing them, until there remain but one unknown Quantity, (or some Powers thereof) equal with the given Quantities. And all the rest of the unknown Quantities concern'd in the Question, are cast off or vanished; which indeed is the chiefest business required in the forming of an Æquation, and therefore ought to be well considered, but no Rules can be prescribed to suit all Cases.

And next, becauſe that in moſt (if not all) Æquations, when firſt they appear out of the covers of the Queſtion, the known and unknown Quantities are intangled and mixt together, on both ſides of the Æquation; it is therefore requiſite ſo to order and clear ſuch Æquations, that one ſide thereof may conſiſt of known Quantities only, and the other ſide of the unknown Quantities, (ſtill keeping them to a juſt equallity,) which work fits the Queſtion for a Solution in Numbers, and is not improperly called *Reduction.*

For the performance thereof, obſerve the following Rules, wherein moſt, or all the parts of *Algebraick Arithmetick*, are concern'd, or imployed.

By Addition.

Rule 1. WHen any Quantity is connected with the Sign —, it is added, by caſting away the Sign —, and transferring the Quantity to the other ſide of the Æquation, with the Sign +

Thus,	1	$a - b - 15 = 60$
$1 + 15$	2	$a - b = 75$
$2 + b$	3	$a = 75 + b$

Again,	1	$aa - bc - d = cc - ba$
$1 + ba$	2	$aa + ba - bc + d = cc$
$2 + bc$	3	$aa + ba - d = cc + bc$
$3 + d$	4	$aa + ba = cc + bc + d$

By Subſtraction.

Rule 2. When any Quantity is Connected with the Sign +, it is to be Subſtracted, by caſting away the Sign

Sign +, and transfering the Quantity to the other ſide of the Æquation, with the Sign —.

Thus,	1	$a + b + c = d$
$2 - c$	2	$a + b = d - c$
$1 - b$	3	$a = d - c - b$

Theſe Two Rules are called Tranſpoſition of Quantities.

By Multiplication.

Rule 3. By this Rule, part, (or the whole) being in Fractions, are brought into whole Quantities.

Thus,	1	$a + b = \frac{dd}{a - b}$
$1 \times a - b$	2	$aa - bb = dd$
$2 + bb$	3	$aa = dd + bb$

By Diviſion.

Rule 4. When there is any Quantity known or unknown concerned in every Member of an Æquation, Divide the whole Æquation by ſuch Quantity, and it will clear one Member thereof (at leaſt) and bring down the reſt.

Thus,	1	$aab + bca = bd$
$1 \div b$	2	$aa + ca = d$

Again,	1	$aab + ca = da$
$1 \div a$	2	$ab + c = d$
$2 - c$	3	$ab = d - c$
$3 \div b$	4	$a = \frac{d - c}{b}$

Again,	1	$baaa + bbaa - ca = da$
$1 + c$	2	$baa + bba - c = d$
$2 + c$	3	$baa + bba = d + c$
$3 \div b$	4	$aa + ba = \frac{d + c}{b}$

By Involution.

Rule 5. When there is an Æquation between like Surds, (or Radical Signs) cast away the Radical Signs, and the Quantities are thereby reduced to Rational ones.

Thus,	1	$aa\sqrt{b} = b\sqrt{da}$
$1 \circlearrowright 2$	2	$aaaab = bbda$
$2 \div ab$	3	$aaa = bd$

Again,	1	$b\sqrt[3]{aa} = \frac{dc}{b}$
$1 \circlearrowright 3$	2	$bbbaa = \frac{dddccc}{bbb}$
$2 \div b^3$	3	$aa = \frac{dddccc}{bbbbbb}$

Also,	1	$\sqrt[2]{\frac{baa + 3ba - d}{4}} = \frac{b + d}{2}$
$1 \circlearrowright 2$	2	$\frac{baa + 3ba - d}{4} = \frac{bb + 2bd + dd}{4}$
2×4	3	$baa + 3ba - d = bb + 2bd + dd$
$3 + d$	4	$baa + 3ba = bb + 2bd + dd + d$
$4 \div b$	5	$aa + 3ba = \frac{bb + 2bd + dd + d}{b}$

By

By Evolution.

Rule 6. By this Rule Æquations are oft brought down to lower Powers, thus,

	1	$aa + 2ba + bb = d$
1 ω 2	2	$a + b = d$

But if the Æquation had been $aa + 2ba = d$, which is an imperfect Square, and wants the Square of bb to fill it up, before it can be brought down as above; such Operation Dr. *Pell* calls, *Compleating the Square*; and when it is performed, useth this Sign $c\,\square$ put in the Margin.

As,	1	$aa + 2ba = d$
1, $c\,\square$	2	$aa + 2ba + bb = d + bb$
2 ω 2	3	$a + b = \sqrt{d + bb}$
3 — b	4	$a = \sqrt{d + bb} - b.$

Suppose,	1	$aa - ba = d$
1, $c\,\square$	2	$aa - ba + \frac{1}{4}bb = d + \frac{1}{4}bb$
2 ω 2	3	$a - \frac{1}{2}ba = \sqrt{d + \frac{1}{4}bb}$
3 + $\frac{1}{2}b$	4	$a = \sqrt{d + \frac{1}{4}bb} + \frac{1}{2}b.$

In these it is evident, that the last place of bb, or that of $\frac{1}{4}bb$, is produced, by involving half the Coefficient b, of the Middle place.

§. When the Æquations be thus cleared or reduced, then are they fit to be resolved into Numbers, and are either Pure and Simple, or Mixt and Adfected. Pure and Simple Æquations, are such as have the unknown Quantity, (a) at one side thereof,

As, $a = b + c$, or $a + b = d$, &c.

alſo when there is only ſome Power of (a,)

As $aa = b$, and $aaa = c$, or $aaaa = d$, &c.

Mixt and Adfected, are ſuch as have the unknown Quantities at one ſide of the Æquation, in different Degrees, or Powers thereof,

As $aa + ba = d$, and $aa - ba = d$, or $ba - aa = d$.

theſe are called the Three Forms of Square Adfected Æquations.

Again, $aaa + ba = c$, and $aaa - ba = c$, or $ba - aaa = c$,

theſe are called the Three Forms of Cubick Adfected Æquations.

Infinite other Varieties there are, ſuch

As $aaa + baa + ca = d$, and $aaa - baa = d$.
Or $aaaa + baaa + caa - da = f$, &c.

How theſe, or any other, of all Degrees, or howſoever Mixt and Adfected, may be eaſily reſolved into Numbers, ſhall be effectually ſhewed further on, when we come to the Reſolving of ſuch Problems as will produce the ſame; taking each in their Order. *viz.* Firſt, I will ſhew how to Reſolve, (or Extract the Roots of) all pure Æquations; next the Three Forms of Square Adfected Æquations, the like in Adfected Cubicks, and ſo on to others of different degrees.

But before we proceed to that part, it will be convenient to inſert a few eaſie Queſtions in Numbers, thereby to explain, and (in part) ſhew the uſe of the foregoing Rules. The which will be as an Introduction to the ſucceeding Problems.

CHAP.

CHAP. VII.

Containing several easie Questions in Numbers fit for a Learners Practice.

Quest. 1. THere be Two unknown, and unequal Numbers, whose Sum is $53 = s$, and their Difference is $21 = d$; What are those Numbers?

For the Greater put a, and for the Lesser put e.

Then according to the Question, they will stand in their Substitutes thus,

		Quantities.	Numbers.
	1	$a + e = s$	$= 53$
	2	$a - e = d$	$= 21$
$1 + 2$	3	$2a = s + d$	$= 74$
$3 \div \bar{2}$	4	$a = \frac{s + d}{2}$	$= \frac{74}{2} = 37$
$1 - 2$	5	$2e = s - d$	$= 32$
$5 \div \bar{2}$	6	$e = \frac{s - d}{2}$.	$= 16$

Hence it appears, that 37, and 16, are the Two Numbers sought.

Note, When any Number is placed in the Margin, with a Line over it, as $\bar{2}$ in the Fourth and, Sixth Steps, such is an absolute Number, and hath no relation to those Numbers in the small Column.

Here you may perceive, that the first Work is to cast off one of the Unknown Quantities; and because the Quantity e, hath both Signs, $+$ and $-$ prefixt to it, therefore by the Rule of *Addition*, it may be vanished, as appears by the Work of the Third Step.

Quest.

Quest. 2. There be Two Numbers (as before) whose Sum is $56 = s$, and the Greater (a) divided by the Lesser (e) the Quotient is $2,5 = q$, What are the Numbers? a, and e.

	1	$a + e = s$	by the Questions.
	2	$a \div e$, or $\frac{a}{e} = q$	
$1 - a$	3	$e = s - a$	
$2 \times e$	4	$qe = a$	
$4 \div q$	5	$e = \frac{a}{q}$	
3, 5,	6	$s - a = \frac{a}{q}$	
$2 \times q$	7	$qs - qa = a$	
$7 + qa$	8	$qa + a = qs$	
$8 \div \overline{q+1}$	9	$a = \frac{qs}{q+1}$ for $\overline{q+1} \times a = qa + 1a$	
$1 - 9$	10	$e = s - \frac{qs}{q+1} = \frac{s}{q+1}$.	

For an Explanation, take the same in Numbers.

	1	$a + e = 56$
	2	$a \div e$, or $\frac{a}{e} = 2,5$
$1 - a$	3	$e = 56 - a$
$2 \times e$	4	$2,5e = a$
$4 \div \overline{2,5}$	5	$e = \frac{a}{2,5} = 56 - a$
$5 + a$	6	$a + \frac{a}{2,5} = 56$
$5 \times \overline{2,5}$	7	$2,5a + a = 140$
$7 \div$	8	$a = \frac{140}{3,5} = 40$ That is, by $2,5 + 1$
3,	9	$e = 56 - 40 = 16$

Quest.

Quest. 3. Let there be given the Difference of the aforesaid Two Numbers, to wit, $24 = d$, and the Quotient, $2,5 = q$, from thence to find out the Two Numbers.

	1	$a - e = d$ } by the Question,
	2	$\frac{a}{e} = q$ }
1 + e	3	$a = d + e$
2 × e	4	$a = qe$
3, 4,	5	$qe = d + e$
5 − e	6	$qe - e = d$
8 ÷	7	$e = \frac{d}{q - 1}$
1 + 7	8	$a = d + \frac{d}{q - 1} = \frac{qd}{q - 1}$.

The reducing of this into Numbers, is easie, (the foregoing Questions considered.)

Quest. 4. There be Two Numbers, their Sum is $56 = s$, and the Difference of their Squares is $1344 = x$; What are they Numbers?

	1	$a + e = s$ } by the Question,
	2	$aa - ee = x$ }
2 ÷ 1	3	$a - e = \frac{x}{s}$ for $a + e \times a - e = aa - ee$
1 + 3	4	$2a = s + \frac{x}{s} = \frac{ss + x}{s}$
4 ÷ 2	5	$a = \frac{ss + x}{2s}$
1 − 3	6	$2e = s - \frac{x}{s} = \frac{ss - x}{s}$
6 ÷ 2	7	$e = \frac{ss - x}{2s}$.

Hence (a) is found equal to (40, and (e) equal to (16) by the 5th and 7th Steps.

Quest.

Quest, 5. Two Men have each a Sum of Money, the one hath Four times as much as the other; both their Sums put together, will not make 100 *l.* but if they be doubled, and from that double be taken 30 *l*, the remainder will be twice as much above 100 *l.* as before they wanted of 100 *l.* How much hath each Man?

Put *a* for the Greater, and *e* for the Lesser.

Then	1	$a = 4e$	*d*, signifieth the difference their Sums wanted of 100 *l.*
	2	$a + e = 100 - d$	
And	3	$2a + 2e - 30 = 100 + 2d$, by the Question.	
$2 \times \overline{2}$	4	$2a + 2e = 200 - 2d$	
$3 + 4$	5	$4a + 4e - 30 = 300$	
$5 + \overline{30}$	6	$4a + 4e = 330$	
1, 6,	7	$5a = 330$	
$7 \div \overline{5}$	8	$a = \frac{330}{5} = 66$	
1, 8,	9	$4e = 66$	
$9 \div \overline{4}$	10	$e = \frac{66}{4} = 16{,}5$	

Hence it is found, that one Man had 66 *l.* and the other 16 *l.* 10 *s.* which Two Sums will answer the Question in each particular.

Note, That as *e*, in the first Question, was cast off by *Addition*, (for the Reasons there mentioned,) so in the Second, the like was done by *Multiplication* and *Division*; in the Third, by *Multiplication* and *Addition*; in the Fourth, by *Division* and *Addition*; and in this Question by *Multiplication* and *Equallity*.

Now if the Reason of these be considered, and once understood, the whole business of forming any Æquation, will be easie; for the only Mystery therein, lies in casting off all the unknown Qualities but one, (of which, see *Pag.* 27.)

Quest.

Queſt. 6. Three Men part their Money in this Manner; the Firſt gives to the Second and Third, as much as they have about them, the Second gives to the Firſt and Third as much as now they have, and the Third doth the like to the Firſt and Second; after they had done thus, each Man had 20 Shillings. The Queſtion is, what each Man had at firſt?

For the Firſt Man's Sum put a, for the Second e, and for the Third y.

Then	1	$a - e - y$	is what the Firſt hath left after his Gift.
	2	$2e$	is what the Second and Third have, after the Firſt Man's Gift.
	3	$2y$	
$2 - 1 - 3$	4	$3e - a - y$	
$1 \times \overline{2}$	5	$2a - 2e - 2y$	the Firſt and Third doubled by the Second.
$3 \times \overline{2}$	6	$4y$	
$6 - 4 - 5$	7	$7y - a - e$	and ſo much each hath at laſt, *viz.* $= 20$.
$5 \times \overline{2}$	8	$4a - 4e - 4y$	
$4 \times \overline{2}$	9	$6e - 2a - 2y$	
That is	10	$7y - a - e = 20$	
	11	$4a - 4e - 4y = 20$	
	12	$6e - 2a - 2y = 20$	
$11 \div \overline{4}$	13	$a - e - y = 5$	
$12 \div \overline{2}$	14	$3e - a - y = 10$	
$13 + 14$	15	$2e - 2y = 15$	
$10 + 13$	16	$6y - 2e = 25$	
$15 + 16$	17	$4y = 40$	
$17 \div \overline{4}$	18	$y = 10$	
$15 + 2y$	19	$2e = 15 + 2y = 35$	
$19 \div \overline{2}$	20	$e = \frac{35}{2} = 17{,}5$	
13, 18, 20,	21	$a = 32{,}5$	

Anſw. The Firſt Man had 32 Shillings, 6 Pence, the Second had 17 Shillings, 6 Pence, and the Third had 10 Shillings.

Quest. 7. There be Three Numbers with these Properties, *viz.* if to the First and Second, be added half the Third, that Sum will be $= s$, (suppose 46, or any Number at pleasure,) and if one Third part of the First be added to the Second and Third, that Sum $= s$, as before; also if a Fourth part of the Second, be added to the First and Third, that Sum will be $= s$, as was the others. What are these Numbers?

For they three Numbers, put a, e, and y.

Then	1	$a + e + \frac{1}{2}y = s$	stated according to the Question.
	2	$\frac{1}{3}a + e + y = s$	
	3	$a + \frac{1}{4}e + y = s$	
$1 \times \overline{2}$	4	$2a + 2e + y = 2s$	Thus all are brought out of their Fractions
$2 \times \overline{3}$	5	$a + 3e + 3y = 3s$	
$3 \times \overline{4}$	6	$4a + e + 4y = 4s$	
$4 + 6$	7	$6a + 3e + 5y = 6s$	
$7 - 5$	8	$5a + 2y = 3s$	
$8 - 5a$	9	$2y = 3s - 5a$	
$9 \div \overline{2}$	10	$y = \frac{3s - 5a}{2}$	
$4 + 5$	11	$3a + 5e + 4y = 5s$	
$11 - 6$	12	$-a + 4e = s$	
$12 + a$	13	$4e = s + a$	
$13 \div \overline{4}$	14	$e = \frac{s + a}{4}$	
$10 + 14$	15	$e + y = \frac{3s - 5a}{2} + \frac{s + a}{4} = \frac{7s - 9a}{4}$	
$15 \times \overline{3}$	16	$3e + 3y = \frac{21s - 27a}{4}$	
5, 16,	17	$a + \frac{21s - 27a}{4} = 3s$	
$17 \times \overline{4}$	18	$4a + 21s - 27a = 12s$:	
18 $\pm$	19	$23a = 9s$, if (s) be put $= 46$, then	
$19 \div \overline{23}$	20	$a = \frac{9s}{23} = 18$, and $e = 16$ $y = 12$.	

Quest.

Quest. 8. There be Four Numbers, their Sum is $= s$, (any Number at pleasure, suppose 100) and if to the First be added $b = 7$, (or any Number,) from the Second be Subducted b, the Third Multiplied by b, it's required that such Sum, Difference, Product and Quotient be equal to each other. What are the Four Numbers?

For the First put a, for the Second e, for the Third y, and for the Fourth u.

Then	1	$a + e + y + u = s$
and	2	$a + b = e - b$
	3	$yb = \frac{u}{b} = a + b$

according to the Proposals in the Question.

$2 + b$	4	$a + 2b = e$
$3 \div b$	5	$y = \frac{a + b}{b}$
3,	6	$\frac{u}{b} = a + b$
$6 \times b$	7	$u = ba + bb$
1,4,5,6,	8	$a + a + 2b + \frac{a + b}{b} + ba + bb = s$
8,	9	$bba + 2ba + a = bs - b^3 - 2bb - b$
$9 \div$	10	$a = bs - bbb - 2bb - b \div bb + 2b + 1$
		Suppose $s = 100$, and $b = 7$
then		$\frac{bs - bbb - 2bb - b = 252}{bb - 2b + 1 = 64} = 3,9375 = a$
4,	11	$a + 2b = 17,9375 = e$
5,	12	$\frac{a + b}{b} = \frac{10,9375}{7} = 1,5625 = y$
7,	13	$ba + bb = 76,5625 = u$.

Answ. The First Number is 3,9375, the Second is 17,9375, the Third is 1,5625, and the Fourth is 76,5615.

 I have

I have made choice of these few plain and familiar Questions, purely for the Young Learner's Exercise, and have placed them in such Order, as may lead him gradually into this most excellent Art of Æquations. I have likewise trased them from Step to Step, thereby to explain and render each Process easie, (even to the meanest Capacity,) which I doubt not but with a little Consideration, will easily be understood; and then he may proceed to the Resolving others, of a different and more difficult Nature, of which there are great varieties in several Authors, to whom I refer those that are desirous of such Speculations; and proceed, (according to my Design) of Extracting the Roots (in Numbers) of all pure Æquations; then to the Resolving of such Problems only, as will raise different Degrees of Mixt or Adfected Æquations; and there shew how the same may be resolved into Numbers.

Which last Work hath heretofore been a Business of great Difficulty and Labour; as the Studious in this Art no doubt have experienced. But the same is now rendered much easier, and made practicable, by a new Method of Converging Series, invented by the Learned and Ingenious Mr. *Joseph Raphson*, Fellow of the *Royal Society*; and Published in a *Latine* Treatise, by the Title of *Analysis Æquationum Universalis*, Printed 1690; wherein he hath given (and demonstrated) general Theorems for the Solving (or Extracting the Roots) of all manner of Æquations, howsoever Mixt or Adfected.

I shall here give a small Abstract thereof, and refer the more inquisitive Reader to the Book it self; where (I doubt not, but) he will meet with ample satisfaction.

CHAP.

CHAP. VIII.

An Abstract of Mr. Raphson's *Method of* Converging Series.

IN this Abstract, I shall pass over the Author's Demonstration, giving only Two Examples of the Method, explaining the same; then insert a few of his Theorems, with some Remarks upon the whole, and so conclude it.

Example in the Square Root.

Viz. $aa = c$ { Any Given Number whose Square Root is required.

What is the Value of the Root a?

Let the Root a, be supposed to be divided into Two Parts, to express each of which, put or substitute Two Letters, *viz.* $g + x = a$;

Then is, $gg + 2gx + xx = aa = c$.

He rejecteth the power of x, (*viz.* xx,) and then it becomes $gg + 2gx = c$; which is $2gx = c - gg$, and from hence ariseth this

$$\textit{Theorem}, \quad x = \frac{c - gg}{2g}$$

Note, The Number signified by g, may be taken at pleasure; but if g, be made the first Root, or Single side of the Given Square, the sooner will the other Member, (or Converging part x,) Converge to the true Root.

To explain what is here ſaid, take this Example, $aa = c = 2$. *As in* Analyſis Æquationum, *Pag* 5. take the Firſt $g = 1$.

$$\text{Then} \left\{ \begin{array}{r} c = 2 \\ -gg = \underline{1} \end{array} \right\} = 1$$

$$\text{And } 2g = 2) \underline{\quad 1{,}0 \quad} (,5 = x$$

To the Firſt $g = 1$, add $,5 = x$

$$\begin{array}{r} + \underline{,5} \end{array}$$

the new $g = 1{,}5$ for a Second Operation.

$$\text{Then} \left\{ \begin{array}{r} c = 2 \\ -gg = \underline{2{,}25} \end{array} \right\} = -\,0{,}25$$

$$\text{And } 2g = 3) \underline{\quad -\,0{,}25 \quad} (-,083 = x$$

Laſt $g = 1{,}5$ from which muſt now be taken the Converging Number x, becauſe it hath the Sign — prefixt to it.

$$\begin{array}{r} 1{,}5 \\ -\ \underline{,083} \end{array}$$

the new $g = 1{,}417$ for a third Operation.

$$\begin{array}{r} \text{Then } c = 2 \\ -\underline{gg = 2{,}007889} \end{array}$$

$$\text{and } 2g = 2{,}834) \underline{\quad -\ ,007889 \quad} (-,002783 = x$$

$$\begin{array}{r} 1{,}417 \\ -\ \underline{,002783} \\ a = 1{,}414217 \end{array}$$

Or if more accuracy be required, it may be called a New g, for a Fourth Operation; and by repeating the Operations, you may have as many Places in the Root as you pleaſe.

Next

Next for the Extraction of the Cube Root, *viz.*

$$aaa = d,$$

put $g + x = a$, as before in the Square.

Then is, $ggg + 3ggx + 3gxx + xxx = aaa$.

And if the Powers of x, viz. $3gxx + xxx$, be rejected, or cast off, as before in the Square, (the which he doth in all Æquations) then it will become

$$ggg + 3ggx = aaa = d, \text{ and } 3ggx = d - ggg,$$

from hence will arise this *Theorem* $x = \dfrac{d - ggg}{3gg}$

Let $d = 37945$, as in *Analysis Æquationum*, Pag. 6.

$$3\dot{7}94\dot{5} = d,$$

take the first $g = 3$, and $d = 37$, that is to the first point,

$$\text{Then } \left\{ \begin{array}{r} d = 37 \\ -\ ggg = 27 \end{array} \right\} = 10, \&c.$$

$$\text{and } 3gg = 27)\ \ 100\ \ (03 = x$$

First, $g = 3$, to which add the Converging x,
$\quad\ + 03$
the new $g = 33$ for a Second Operation.

$$\text{Then } \begin{array}{r} d = 37945 \\ -\ ggg = 35937 \end{array}$$

$$\text{and } 3gg = 3267)\ \ 2008,0\ (,61 = x$$

$$\begin{array}{r} 33, \\ +\ ,61 \\ \hline \end{array}$$

the new $g = 33,61$ for a Third Operation.

Then

Then $\begin{cases} d = 37945 \\ -ggg = 37966{,}934881 \end{cases}$

and $3gg = 3388{,}8963)\ -21{,}934881(-{,}006472 = x$

Laſt $g = 33{,}61$ from which muſt now be taken
the $-\ {,}006472 = x$
hence, $a = 33{,}603528$

Or if more exactneſs be required, then it may be called a new g, as before, &c.

I ſhall deſiſt giving more Examples in Numbers, this being ſufficient to ſhew the Method of proceeding in each Æquation; and only inſert a Table or Two of the ſeveral Theorems, as they are recorded in Mr. *Raphſon*'s Book before mentioned.

The Given Powers, whoſe Root is required.	Firſt Theorems.	Second Theorems.
For the Square Root $aa = c$	$x = \frac{c - gg}{2g}$	$a = \frac{c + gg}{2g}$
For the Cube $aaa = d$	$x = \frac{d - ggg}{3gg}$	$a = \frac{d + 2ggg}{3gg}$
For the Biquadr. $aaaa = f$	$x = \frac{f - gggg}{4ggg}$	$a = \frac{f + 3gggg}{4ggg}$
For the Surſolide $a^5 = f$	$x = \frac{f - ggggg}{5gggg}$	$a = \frac{f + 4g^5}{5gggg}$

By what is here ſet down, it is obvious how to proceed for Extracting the Root (by this Method) of any ſimple Power, not only of theſe here inſerted, but of any how high ſoever it be.

A

A Table of Theorems, for the Resolving of Adfected Æquations, *wherein I retain only those with the Converging* (x,) *and have omitted the Second sort, which Converge by the* (a) *it self; such as in the Second sort of the foregoing Table.*

Æquations proposed.	Theorems.
$aa + ba = c$	$x = \frac{c - gg - bg}{2g + b}$
$aa - ba = c$	$x = \frac{c + bg - gg}{2g - b}$
$ba - aa = c$	$x = \frac{c + gg - bg}{b - 2g}$
$aaa + ca = d$	$x = \frac{d - ggg - cg}{3gg + c}$
$aaa - ca = d$	$x = \frac{d + cg - ggg}{3gg - c}$
$ca - aaa = d$	$x = \frac{d + ggg - cg}{c - 3gg}$
$aaa + baa + ca = d$	$x = \frac{d - ggg - bgg - cg}{3gg + 2bg + c}$
$aaa + baa - ca = d$	$x = \frac{d - ggg - bgg + cg}{3gg + 2bg - c}$
$aaa - baa - ca = d$	$x = \frac{d + bgg + cg - ggg}{3gg - 2bg - c}$
$baa + ca - aaa = d$	$x = \frac{d + ggg - bgg - cg}{2bg + c - 3gg}$

This Table is continued in Mr. *Raphson*'s Book to Four or Five Pages more, but for brevity sake I am forced to omit transcribing them.

There

There be ſeveral things obſervable in this Method, amongſt the reſt, theſe are not the leaſt, *viz.* That at each Operation, the Converging Number x, will double the laſt preceeding g, (or Number of Figures in the laſt Root, *eſpecially after the Second Operation*, the Imperfection being only in the laſt Figure of the Root, ſo increaſed, which often proves too large, and therefore conſequently the next Converging Number x, will have the Negative Sign —.

Alſo if there happen to be a miſtake committed in any Operation, ſuch miſtake doth not deſtroy the preceeding work, for the ſame will be rectified (though it be not diſcovered) in the next ſucceeding Operation, unleſs it be very groſs.

Again, it produceth the Roots of all Powers, be they never ſo high, in the ſame manner, and with the ſame exactneſs, as it doth thoſe of a lower Rank, the reſpective Involutions being conſidered, which require to be always of the ſame height with the Given Powers; and the Diviſors of the next Inferiour, or lower Powers.

Theſe high Involutions and large Diviſors, have often cauſed me to wiſh, that ſome means might be uſed of improving this Method, (or ſome lucky Diſcovery made of an other) ſuch as would produce the ſeveral Reſolvends and Diviſors in lower Terms: And being (upon an extraordinary occaſion) imployed in the Calculation of ſeveral Problems, conſiſting of very high Adfected Æquations, made me the more deſirous thereof, and put me upon thoughts, how (if poſſible) to accompliſh the ſame; which if effected, would undoubtedly render the ſeveral Operations more eaſie.

Whilſt I was ruminating upon this, there came into my Mind the following Method, the which hath not only anſwered my Deſire in that particular, of producing Low Reſolvends and Diviſors, but even out-done my Expectation, in cauſing the Root to Converge quicker; for at each Operation, it will in moſt caſes triple the Places of Figures in the laſt preceeding Root; in ſeveral it will do more, and in ſome caſes, the Root Convergeth by a continued Series, without repeating the Operations at all, as will plainly appear in the Practice thereof.

CHAP.

CHAP. IX.

Containing a new and general Method of Extracting the Roots (in Numbers) of all pure Æquations.

I Shall communicate this Method with all the plainneſs (and brevity) I can, and therein ſhew the Proceſs uſed in raiſing and forming the Theorems for the Solution of each Æquation, as they come in their Order.

Now becauſe there is required an Involution of the Increaſed Root, at the repeating of ſeveral Operations, (tho not ſo high as the Given Power,) I will here inſert an eaſie Method of Compoſing the Square of an increaſed Root, by having the Square and its Root, of one part thereof, with the increaſing Numbers of the other part of the Root, performed without involving the whole Root ſo increaſed.

Root =	7 4 9 3 8 0 5 2
Square of 7 =	49 | 16
	28
	28
Square of 74 =	5476 | 8109
	2247
	2247
	666
	666
Square of 7493 =	56145049 | 64002504
	14987610
	14987610
	3746900
	3746900
	59944
	59944
Square of 74938052 =	5615711637554704

This

This is so obvious, I suppose there needs no Explanation thereof: *The difference betwixt it and the common Method, I leave to the consideration of the Ingenious*

From the foregoing Genesis, or Composition of the Square, it is evident, that for each single Figure, (or Cypher in the Root, there will arise Two places of Figures in the Square; and from the like Composition of the Cube, it would be as evident there will arise Three, and in the Biquadrate Four, *&c.*

That is, there will arise so many Places of Figures in the Involved Number, for one in the Root, as the Index of such Power denotes.

Therefore when any Number is proposed to have its Root Extracted, the preparative work is to consider the Index of the Required Root, and to allow so many places of Figures in the Given Number, for each single Figure in the Root, as such Index shall denote; which Account must always take its beginning from the place of Unity, and ascend towards the Left-hand, if the Given Number be Integers, or descend towards the Right in Decimal parts.

Of the Index of any Root, see *Page* 5, and such Index is the same with that of the Power, *viz.*

$$\left.\begin{array}{l} aa = b \\ aaa = c \\ aaaa = d \\ aaaaa = f \end{array}\right\} \text{ is } \left\{\begin{array}{l} a^2 = b \\ a^3 = c \\ a^4 = d \\ a^5 = f \end{array}\right\} \text{ then } \left\{\begin{array}{l} a = \sqrt[2]{b} \\ a = \sqrt[3]{b} \\ a = \sqrt[4]{d} \\ a = \sqrt[5]{f} \end{array}\right.$$

And from hence is deduced the Method of distinguishing (or rather parting) the Given Numbers with Points, set over, or under their proper Figures.

For Example, Suppose any Number given,

As, 56430145789,723

out of which, if it be required to Extract any of these

Roots,

Roots, it muſt be pointed (according to the foregoing Conſiderations) in this manner, that is, for the

Square Root,	$\dot{5}6\dot{4}3\dot{0}1\dot{4}5\dot{7}8\dot{9},7\dot{2}3\dot{0}$
Cube Root,	$5\dot{6}43\dot{0}14\dot{5}78\dot{9},72\dot{3}$
Biquadrate Root,	$56\dot{4}3014\dot{5}78\dot{9},723\dot{0}$
Surſolide Root,	$\dot{5}6430\dot{1}4578\dot{9},7230\dot{0}$

And by theſe Points is known how many places of Figures there will be in the Root ſought, for ſo many as are the Points, ſo many muſt be the Places of Figures in the Root. Now whether they be Integers or Decimals, is eaſily determined, by the Places of the Points over each. Theſe premiſed, we may proceed to Examples in each.

Note, that in all the Theorems for reſolving of Æquations (or extracting of Roots) by the following Method, I make uſe of theſe Letters (*G*,) (*r*,) (*e*,) (*D*,) to repreſent each diſtinct part thereof, *viz.*

G, Always denotes the Given Reſolvend, or known ſide of the Æquation.

r. Repreſents the Root firſt taken, and when increaſed, *&c.*

e, Repreſents the Converging part of the Root; by which, *r*, is either increaſed or diminiſhed, according as its Sign denotes, to wit, $+e$, or $-e$.

D, Is put for the Dividend, which is produced from *G*, being divided; and leſſened by *r*, (into the Coefficients of Affected Æquations) according as the Root requires; all which will plainly appear, and be eaſily underſtood, as we proceed on in the Examples.

I. *For the Extracting of the Square Root*, viz. $aa = G$. What is the value of a?

In this Method (as in all others whatſoever) we ſuppoſe the Root a, to be Divided into Two parts, to

 expreſs

express which, put the Two Lettters before mentioned.

That is,		
	1	$r + e = a$
$1 \circledS 2$	2	$rr + 2re + ee = aa = G.$
$2 \div 2$	3	$\frac{1}{2}rr + re + \frac{1}{2}ee = \frac{1}{2}G$
$3 - \frac{1}{2}rr$	4	$re + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr = D.$

And from hence will arise this

$$\text{Theorem} \left\{ \frac{D}{r + \frac{1}{2}e} = e. \right.$$

First take an Example of a true Square Number.

$$\text{Let } aa = 4196837089 = G$$

$r = 6,$ That is, the First Single Root, (or side) to 41

$$2098418544{,}5 = \tfrac{1}{2}G$$

$$-\tfrac{1}{2}rr = -18$$

$$r = 6 \qquad) \; 298418544{,}5 = D \; (4 = e$$

$$+ e = 4 \qquad 248 = re + \tfrac{1}{2}ee$$

$$r + e = 6473 \qquad 50418 \quad (7 = e$$

$$45045 = re + \tfrac{1}{2}ee$$

but $r + e = a$ $\qquad 53735 \quad (8 = e$

therefore $\qquad 51792 = re + \tfrac{1}{2}ee$

$a = 64783$ $\qquad 194344{,}5 \quad (3 = e$

$$194344{,}5 = re + \tfrac{1}{2}ee$$

$$(0{,}0)$$

I presume the whole process of this is plain, however for the benefit of such as do not understand an Algebraick Theorem, let them take it in words thus.

First

First point the Resolvend (or Given Number) as before taught; then take the greatest Square to the first Point thereof, out of the Resolvend to that Point; after which, half the remainder of the Resolvend, and point it anew; then make the Root of that Square so taken, a Divisor; inquiring how oft it may be found in the new Dividend, to the next Figure forward, reserving that Figure under the next Point, for the half Square of the Quotient Figure; which being found, and so placed, Multiply the Divisor by that Quotient, adding in the Tens, (if any arose from the half Square of the Quotient) as in plain Division, &c. Then annex the Quotient to the last Divisor, thereby making a new Divisor, with which proceed in all respects as with the last; and so on, until all be finished.

If any seeming Difficulty appear, 'tis only in the true placing of the half Square of the Quotient Figure when it proves an odd Number; in that case bring down one Figure more of the next Period; under which, place the odd 5, which will always arise from the halving of the Square of an odd Number.

As for instance, Suppose 7, the Square thereof is 49, the half of which is 24,5 to be placed as in the Second Operation of the last Example.

Note, That if the Given Number be a *Surd*, the Root *r*, will become a continued Series, *ad infinitum*; if it be still increased with the Converging Number *e*.

But if the Number of Places (in the Root of a *Surd* Number) be limited, the Root *r*, needs not be increased with *e*, to the last Operation of the whole Process; for the Work may be abreviated, as in this Example. Suppose the Square Root of 3, (a *Surd* Number) were required to Sixteen places of Figures therein.

Æquation $aa = G$, and $G = 3$.

$r = 1,$ $\quad 1,5 = \frac{1}{2}G$

$+e = ,7$ $\quad -0,5 = \frac{1}{2}rr$

$r+e = 1,7320508)\ 1,0000000000000000 = D$

945 $= re + \frac{1}{2}ee$ $\quad$ (,7 $= e$

550 $\quad$ (3 $= e$

5145 $= re + \frac{1}{2}ee$

3550 $\quad$ (2 $= e$

3462 $= re + \frac{1}{2}ee$

880000 $\quad$ (05 $= e$

8660125 $= re + \frac{1}{2}ee$

139875000 $\quad$ (08 $= e$

138564032 $= re + \frac{1}{2}ee$

1310968

Having thus increaſed the Root to 8, or 9, Places by the Converging *e*, the reſt may be obtained by plain *Diviſion*, making the Root ſo found, a Diviſor; but I alſo contract plain Diviſion, by omitting, (or pricking off) one Figure of the Diviſor at each Operation; and then the Work will ſtand thus,

$r = 1,7320508)$ 131096800 (07568877 $= e$

121243556

9853244

8660250

1192994

1039230

153764

138560

15204

13856

1348

1211

137

119

&c.

This Method of Contracting the Diviſor, is of ſingular uſe upon ſeveral Occaſions, where it may be admitted.

And what is here done as to this Abridgment, may in like manner be performed in the Cube, Biquadrate, &c.

Laſt $r = 1,7320508$

$+e$ $\quad$ 07568877

$a = 1,732050807568877$ as was required.

II. Of

II. *Of Extracting the Cube Root.*

Æquation $aaa = G$. To find the value of a?

Put	1	$r + e = a$
1 ⓢ 3	2	$rrr + 3rre + 3ree + eee = aaa = G$
$2 \div 3$	3	$\frac{1}{3}rrr + rre + ree + \frac{1}{3}eee = G$
$3 \div r$	4	$\frac{1}{3}rr + re + ee + \frac{1}{3}\frac{eee}{r} = \frac{1}{3}G \div r.$

Let $\frac{1}{3}\,eee \div r$ be rejected, or cast off, as being of small Value, then it will become

$$\tfrac{1}{3}rr + re + ee = \tfrac{1}{3}G \div r.$$

and thence will $re + ee = \frac{\frac{1}{3}G}{r} - \frac{1}{3}rr = D.$

Hence ariseth this

Theorem $\left\{ \frac{D}{r+e} = e. \right.$

Let $\dot{3}79\dot{4}5 = G$, (as in *Pag* 43 of this Tract.)
then $r = 30$, and $12648{,}333 = \frac{1}{3}G$

```text
                  421,6111   = 1/3 G ÷ r
  r = 30         —300,     = 1/3 rr
 +e =  3          121,6111 = D          (3 = e
r + e 33,6          99     = re + ee
                    2261                (,6 = e
                    2016   = re + ee

New r = 33,6   376,43849206 = 1/3 G ÷ r
             — 376,32       = 1/3 rr
r+e=33,6035262) ,11849206 = D      (,003 = e
                 100809   = re + ee
                 1768306           (5 = e
                 1680175  = re + ee
a=33,6035262, &c.  8813100          (2 = e
                 6720604  = re + ee
                 209249600          (6 = e
                 201621156
                   762844400        (2 = e
                   672070524
```

Here you have the Cube Root Extracted to Nine Places, at Two easie Operations, the Defect being but Unity, in the last Place thereof.

 This

This Method doth alſo Converge with the Negative Sign — if the value of e, happen to be taken too large; but to prevent that, take but one value of e, at the firſt Operation, and but Four at the Second; by ſo doing, the Root will have but Six Places at Two Operations: But then a Third will produce Seventeen Places true; each Operation converging gradually, with the Affirmative Sign (which I take to be beſt.)

Take an Example of a true Cube Number, which I find in my Friend Mr. *William Hunt*'s *Clavis Stereometriæ*, *Page* the 9th, *viz.*

$$aaa = 48627125 = G$$

$$\begin{array}{lll}
n = 300 & 16209041{,}66\ \&c. = \tfrac{1}{3}G & \\
 & 54030{,}1388\ \&c. = \tfrac{1}{3}G \div r & \\
 & \underline{-\ 30000, = \tfrac{1}{3}rr} & \\
r + e = 366, & 24030{,}1388 = D & (66 = e \\
 & \underline{216 = re + ee} & \\
 & 2430 & \\
\text{New } r = 366 & \underline{2196} = re + ee. &
\end{array}$$

But as was ſaid above, I take but one place, then

$$\begin{array}{lll}
r = 360 & \text{and } 45025{,}1157 = \tfrac{1}{3}G \div r & \\
 & \underline{-\ 43200, = \tfrac{1}{3}rr} & \\
r+e = 365{,}0003 & 1825{,}1157 = D & \\
 & \underline{1825, = re + ee} & (5{,}0003 = e \\
 & 0{,}11570000 & \\
\text{hence } a = 365{,}0003 & 10150009 = re + ee &
\end{array}$$

The Root is really but 365, this overplus of 0003, would at the next Operation vaniſh. And from this Example, 'tis apparent, that Four Places will truly ariſe in the Root, at the Second Operation.

The work of theſe Two laſt Examples, (I preſume) is ſo plain and clear, that its needleſs to expreſs the Proceſs thereof in Words: Alſo from hence it's as plain, that the Extraction of the Cube Root, (heretofore ſo very difficult) is by this Method rendered eaſier, than that of the Square; as it's uſually performed.

III. Of

III. *Of Extracting the Biquadrate Root*, viz. $aaaa = G$. What is the value of a?

Let $aaaa = G = 2741583974$, and $r + e = a$

Then	1	$rrrr + 4rrre + 6rree + 4reee + eeee = G$
$1 \div 4$	2	$\frac{1}{4}rrrr + rrre + 1\frac{1}{2}rree + reee + \frac{1}{4}eeee = \frac{1}{4}G$
$2 \div rr$	3	$\frac{1}{4}rr + re + 1\frac{1}{2}ee + \frac{eee}{r} + \frac{\frac{1}{4}eee}{rr} = \frac{1}{4}G \div rr$

Let $eee \div r$, and $\frac{1}{4}eeee \div rr$ be rejected, or cast off; (as before,) then it will become

$$\tfrac{1}{4}rr + re + 1\tfrac{1}{2}ee = \tfrac{1}{4}G \div rr$$

and consequently $re + 1\frac{1}{2}ee = \frac{\frac{1}{4}G}{rr} - \frac{1}{4}rr = D.$

From hence ariseth this

Theorem $\left\{ \dfrac{D}{r + 1\frac{1}{2}e} = e \right.$

First $r = 200$	$2741583974 = G$
	$685395993,5 = \frac{1}{4}G$
	$17134,899$ &c. $= \frac{1}{4}G \div rr$
$r + e = 229$	$-\ 10000 = \frac{1}{4}rr$
$+ \frac{1}{2}e = 14,5$	
Divisor 23	$7134,89$ &c. $= D\ (29 = e$
Divisor $233,5$	$46 = re + 1\frac{1}{2}ee$
	$2534,8$
	$2101,5 = re + 1\frac{1}{2}ee.$
New $r = 229,$	$13069,8498 = \frac{1}{4}G \div rr$
$-\ 1\frac{1}{2}e = ,15$	$-\ 13110,25 = \frac{1}{4}rr$
Divisor $228,85$	$-\ 40,4002 = D\ (-,1 = e$
New $r = 228,9$	$22,885 = re - 1\frac{1}{2}ee$
$-\ 1\frac{1}{2}e = ,105,$	$17,51520\ (-7 = e$
Divisor $228,795,$	$16,01565 = re - 1\frac{1}{2}e$
New $r = 228,83$	$1,499550\ (6 = e$
$-\ 1\frac{1}{2}e = ,009$	$1,372926 = re - 1\frac{1}{2}e$
Divisor $228,821$	$1266240\ (5 = e$
	1144105

$r = 229,$
$-\ ,1765$
$a = 228,8235$

In

In this Example, it appears I took the Second $r = 229$ too big, and therefore in the Second Process, the Member e, Convergeth with the Sign — and consequently (in this Case) the Divisor must be $re - 1\frac{1}{2}e$; also at each Division, e, is to be taken from the last r, and the remainder will be the New r.

IV. *Of Extracting the Root of a Surfolide, (or Fifth Power,)* viz. $aaaaa = G$. What's the value of a?

In Numbers $aaaaa = 43467897 = G$.

Put	1	$r + e = a$. Reject all the Powers of e above ee.
then	2	$rrrrr + 5rrrre + 10rrree = G$
$2 \div 5$	3	$\frac{1}{5}rrrrr + rrrre + 2rrree = \frac{1}{5}G$
$3 \div r^3$	4	$\frac{1}{5}rr + re + 2ee = \frac{1}{5}G \div rrr$
$4 - \frac{1}{5}rr$	5	$re + 2ee = \dfrac{\frac{1}{5}G}{rrr} - \frac{1}{5}rr = D$

From hence ariseth this

$$\textit{Theorem;}\quad x = \frac{D}{r + 2ee} = e.$$

First $r = 30$

$$\begin{aligned} 43467897 &= G \\ 8693579{,}4 &= \tfrac{1}{5}G \\ 321{,}96,\&c. &= \tfrac{1}{5}G \div rrr \\ -180, &= \tfrac{1}{5}rr \\ \hline 141{,}96 &= D \quad (3 = e \\ 108 &= re + 2ee \end{aligned}$$

$$\begin{aligned} r + e &= 33 \\ + e &= 3 \\ \hline \text{Divisor}\quad & 36 \end{aligned}$$

$$\begin{aligned} \text{New}\quad r &= 33, \\ + 2e &= 1{,}4 \\ \hline \text{Divisor}\quad & 34{,}4 \\ \text{New } r + e &= 33{,}7009 \\ + e &= {,}0009 \\ \hline \text{Divisor}\quad & 33{,}7018 \end{aligned}$$

$$\begin{aligned} 241{,}91160019 &= \tfrac{1}{5}G \div r^3 \\ -217{,}8 &= \tfrac{1}{5}rr \\ \hline 24{,}11160019 &= D \quad ({,}7 = e \\ 24{,}08 &= re + 2ee \\ \hline {,}03160019 & \quad ({,}0009 = e \\ 3033162 &= re + 2ee \end{aligned}$$

$$r + e = 33{,}7009 = a.$$

But if more Exactness be required, the next Operation will produce 17 places of Figures true.

I

I doubt not, but by these few Examples, the nature of, and manner how to proceed in, this Method is sufficiently cleared; as to the Extraction of Roots out of Simple or pure Æquations, how highly soever they be.

And because there is great care and trouble attends the continued Involutions of a Binomial, (to wit of $r + e$) especially to any considerable height, by reason of the *Unciæ*, (or Numeral Figures, that arise by involving the Quantities) I will here shew, how any Power, of what Degree soever, may be speedily and easily raised: without Involving the Quantities at all. Thus

First make the Index of the highest Power of r, the same to that of the given Power of a; then the next Power of r, (into e,) must have its Index less by Unity, and the Third Power of r (into ee) must have its index less than the First by 2, that is, their Indices decrease in Arithmetical Progression.

Next for obtaining the *Unciæ*, or respective Nnmeral Figures that would arise by a continued Involution of $r + e$, prefix to the Second Power of r, (into e,) the Index of the First (or simple) Power of r. And to the Third Power of r (into ee) prefix half the Product of the first Index, into the Index of the Second, and these will be the true Numbers required.

For the clearing of this, take an Example or Two. Suppose a^9, were given, and you are to raise $r + e$ to the same height (that is, so much thereof as is required in this Method) first set down r^9, to this add r^8e, with the Index of the first prefix'd to it; then it will be $r^9 + 9r^8e$. To these add r^7ee, with half the Product of 9×8, to wit, $9 \times 4 = 36$, and then it will be $r^9 + 9r^8e + 36r^7ee = a^9$, as was required.

Again, suppose a^{18}, were given; to raise the rest as before, $r^{18} + 18re^{17}$, are the Two first Members, and the half of 18×17, that is, $9 \times 17 = 153$, and then they will be $r^{18} + 18r^{17}ee + 153r^{16}ee = a^{18}$.

And this I take to be a new Discovery, having never seen any thing performed in this kind, with the like ease and expedition.

CHAP.

CHAP. X.

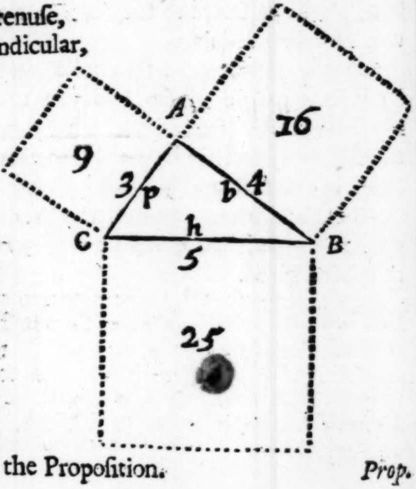
The Resolving of several Geometrical Problems, together with the Solution of their Æquations in Numbers.

FOR the Resolving of *Geometrical Problems,* it will be requisite to have a clear understanding of these Two Propositions of *Euclid*, viz. The 47th of the 1st, and 4th of the 6th Book, of their General, (and excellent) use in *Geometry,* (and consequently in all parts of the *Mathematicks,*) none that hath been conversant therein, but will readily grant; of which *Des Cartes* takes particular notice, in a Lettter of his, cited by Dr. *Pell,* in the before-mentioned Book of *Algebra*, Pag. 65.

And here it may not be amiss (for the Benefit of the Young Learner,) to give a particular account of the aforesaid Propositions.

Prop. I. In all Right-angled Triangles, the Square of the Hypotenuse (or greatest side) is equal to the Sum of the Squares of the Base, and Perpendicular, (or of the other Two sides.) *Eucl.* 1. 47.

Suppose the Triangle *C A B*, to be right-angled at *A*. Then let



$BC = h =$. Hypotenuse,
$AC = p =$. Perpendicular,
$AB = b =$. Base.

And then it is

$$\left.\begin{aligned} hh &= bb + pp \\ bb &= hh - pp \\ pp &= hh - bb \end{aligned}\right\}$$

That is, in Numbers thus; suppose $h = 5$, $b = 4$, and $p = 3$, as in the Scheme. Then is, $hh = 25$, and $bb = 16$, and $pp = 9$, but $16 + 9 = 25$, which makes good the Proposition.

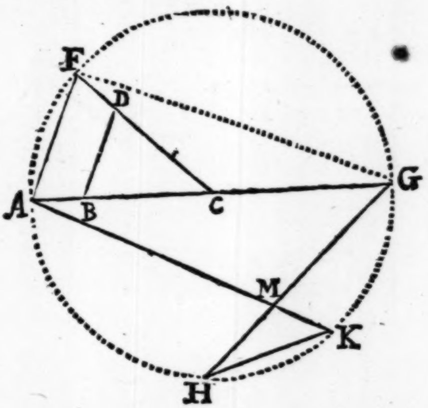
Prop.

Prop. II. The sides of like Triangles, are respectively proportional to each other, *Eucl.* 6. 4.

That is, if there be Two Triangles, (either Right or Oblique) that have each Angle of the one, equal to each Angle of the other, then are those Two Triangles alike, and their respective sides proportional.

Angles are equal, that stand upon, (or have) equal Arches, either both from the Center, or both from the Perifery of a Circle. As for Example, Suppose the Two Triangles to be *ACF*, and *BCD*, having the Angle at *C*, (the Center) common to both, and *BD*, parrallel to *AF*.

Then is the Angle at *D*, equal to that at *F*, and consequently the Angle at *B*, equal to that at *A*. And according to the Proposition, *AC* : *AF* :: *BC* : *BD*. and *FC* : *AC* :: *DC* : *BC*. &c.



Again the Triangles *AMG*, and *HMK* are alike; for the Angles *A*, and *H*, stand both upon the Arch *GK*, and the Angles *G*, and *K*, stand both upon *AH*; also the Angle at *M*, is alike in both. Therefore according to the Proposition,

AG : *HK* :: *AM* : *HM*, and *GM* : *MK* :: *AM* : *MH*, &c.

To these Two, add, that the Angle at the Center, is double to that at the Perifery; that is, the Angle *FCA*, is Double to the Angle *FGA*. *Eucl.* 3. 20.

These

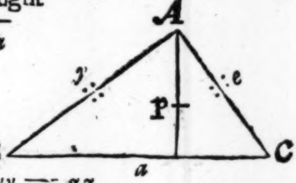
These Propositions being premised, and pretty well understood, the young Algebrist may proceed to the Resolution of Problems, into which I endeavour to introduce him, with some that be easie.

PROB. I.

IN the Right Angle Triangle *BAC*, there is given the Sum of the Sides *AB*, and *AC*; also the Perpendicular *P*, let fall from the Right Angle *A*, upon the Hypotenuse, *BC*. To find the Hypotenuse, and the Sides.

Let the Hypot. : $BC = a$, $AC = e$. $AB = y$.

Then	1	$\left.\begin{array}{r} e + y = z \\ p = p \end{array}\right\}$ given $\left\{\begin{array}{l} z = 48 \\ p = 17 \end{array}\right.$
	2	
	3	$BC = a$, sought
Prop. 1.	4	$ee + yy = aa$
Prop. 2.	5	$y : a :: p : e$
5 ∵	6	$ye = pa$
1 ⊛ 2	7	$ee + 2ye + yy = zz$,
$6 \times \bar{2}$	8	$2ye = 2pa$
4 + 8	9	$ee + 2ye + yy = aa + 2pa$
7, 9,	10	$aa + 2pa = zz$.



This is called the First Form of Square Adfected Æquations, and may be solved, by compleating the Square; (as in *Page* 31.) Then it will become

$$aa + 2pa + pp = zz + yp.$$

and consequently $a + p = \sqrt{zz + pp}$.

Then will $a = \sqrt{zz + pp} - p$

And this Method is easie enough, especially by the Method of Extracting the Square Root. (*Page* 52,) But the same may be readier done by the following Method.

Note,

Note, *When* Prop 1, *or* Prop. 2. *is placed in the Margin, they refer to the Two Propositions before mentioned,* Page 58, *and* 59.

The New found Æquation is $aa + 2pa = zz$.

In Numbers $aa + 34a = 2304 = G$.

Put,	1	$r + e = a$
1 ② 2	2	$rr + 2re + ee = aa$
1 × 2p	3	$2pr + 2pe = 2pa$
3 + 2	4	$rr + 2pr + 2re + 2pe + ee = G$
4 ÷ 2	5	$\frac{1}{2}rr + pr + re + pe + \frac{1}{2}ee = \frac{1}{2}G$
Then	6	$re + pe + \frac{1}{2}ee = \frac{1}{2}G - \frac{1}{2}rr - pr = D$

From hence ariseth this

Theorem $\left\{ \frac{D}{r + p + \frac{1}{2}e} = e \right.$

$r = 30$	$2304 = G$	and $17 = p$
$rr = 450$ } = −	$1152 = \frac{1}{2}G$	
$pr = 510$ }	$960 = pr + \frac{1}{2}rr$	
$r + p = 47$	$192,00 = D$	(3, = e
$+ \frac{1}{2}e = 1,5$	145.5	
Divisor 48,5	46,5000	(,9 = e
New $r + p = 50,$	45,405	
$+ \frac{1}{2}e = ,45$	1,0950	(2 = e
2 Divisor 50,45	1,0182	
New $r + p = 50,9$	768000	(1 = e
$+ \frac{1}{2}e = ,01$	509205	
Divisor 50,91	258795	(50823 = e
New $r + p = 50,92$	254605	For finding the
$+ \frac{1}{2}e = ,0005$	4190	first r.
Divisor 50,9205	4072	Suppose $r = 4$.
New $r + p = 50,921$ *	118	$rr = 16$
	100	$3r = 12$
*Here I desist forming	18	$28 > 23$
a new Divisor	15	*Ergo*, $r < 4$
First $r = 30$	&c.	

$+ e = 3,92150823$

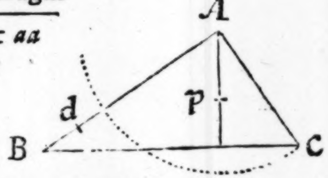
$a = 33,92150823$ This is also a continued Series as the Square Root was.

PROB. II.

SUppose in the Triangle BAC; there were given $d =$ the difference of the Sides AB, and AC. Also the Perpendicular P, let fall from the Right Angle A, upon the Hypotenuse BC. Thence to find the Hypotenuse, &c.

Let $BC = a$. $AC = e$. And $AB = y$. (as before.)

Then	1	$y - e = d$	given $d = 14$
	2	$p = p$	given $p = 20$
	3	$BC = a$ sought.	
Prop. 1.	4	$yy + ee = aa$	
Prop. 2.	5	$y : a :: p : e$	
5 ∵	6	$ye = pa$	
6 × 2	7	$2ye = 2pa$	
1 $\circledS$ 2	8	$yy - 2ye + ee = dd$	
8 + 7	9	$yy + ee = dd + 2pa$	
4, 9,	10	$aa = dd + 2pa$	
Then	11	$aa - 2pa = dd.$	



This is called the Second form of Square Adfected Æquations.

And may be solved by Compleating the Square, as in the first Form, respect had to the Sign —

Then it will become

$$aa - 2pa + pp = dd + pp.$$

And consequently, $a - p = \sqrt{dd + pp}.$

Then will $a = \sqrt{dd + pp}. + p.$

But this Form may be solved (by my Method) as was the last, to wit, by a continued Series.

Æquation

Æquation $aa - 2pa = dd$.

In Numbers, $aa - 40a = 196 = G$,

Put	1	$r + e = a$
1 ⊙ 2	2	$rr + 2re + ee = aa$
1 × 2p	3	$2pr + 2pe = 2pa$
2 — 3	4	$rr + 2re - 2pr - 2pe + ee = G$
4 ÷ 2	5	$\frac{1}{2}rr + re - pr - pe + \frac{1}{2}ee = \frac{1}{2}G$
Then	6	$re - pe + \frac{1}{2}ee = \frac{1}{2}G + pr - \frac{1}{2}rr = D$

From hence ariseth this

Theorem $\left\{ \dfrac{D}{r - p + \frac{1}{2}e} = e. \right.$

$r > 30$, because of $-2pr$. Suppose $r = 30$. Then $rr = 900$, and $2pr = 1200$. But 1200 cannot be taken out of 900, therefore $r > 30$. Suppose $r = 50$, $rr = 2500$, and $2pr = 2000$, then $rr - 2pr = 500$, too much, *Ergo* $r > 30$, but $r > 50$.

Let $r = 40$	$196 = G$	and $20 = p$
$+ pr = 800$	$98 = \frac{1}{2}G$	
$- \frac{1}{2}rr = 800$	$00 = pr - \frac{1}{2}rr$	
$r - p = 20,$	$98, = D$	$(4 = e$
$+ \frac{1}{2}e = 2,$	$88,$	
Divisor $22,$	$10,00$	$(,4 = e$
$r - p = 24,$	$9,68$	
$+ \frac{1}{2}e = ,2$	32000	$(1 = e$
$r - p = 24,4$	24405	
$+ \frac{1}{2}e = ,005$	759500	$(3 = e$
$r - p = 24,41$	732345	
$+ \frac{1}{2}e = ,0015$	27155	$(11124 = e$
$r - p = 24,413$ *	24413	
	2742	
	2441	
	301	
	244	
	57	
	48	
	9	
	8	
	&c.	

* Here I desist forming a new Divisor.

First $r = 40$

$+ e = 4,41311124$

$a = 44,41311124$

PROB.

PROB. III.

IN the Right Angle Triangle BCA, the Base (or Side) BA, is given, And the Hypotenuse with the Area, in one Sum; thence to find the Perpendicular AC.

Let $AC = a$, $BC = e$, and $BA = b$, Given.

Then	1	$\frac{1}{2}ba =$ the Area
And	2	$e + \frac{1}{2}ba = z$ the $\left\{\begin{array}{l}\text{Sum of the Hypot.}\\ \text{and Area.}\end{array}\right.$
Prob 1.	3	$ee = bb + aa$
$2 - \frac{1}{2}ba$	4	$e = z - \frac{1}{2}ba$
$4 \circledcirc 2$	5	$ee = zz - zba + \frac{1}{4}bbaa$
3, 5,	6	$bb + aa = zz - zba + \frac{1}{4}bbaa$
$6 \pm$	7	$aa - \frac{1}{4}bbaa + zba = zz - bb$

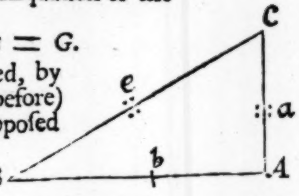
Let $z = 69$, and $b = 7$, then the Æquation in Numbers will be $42{,}93\,a - aa = 418{,}844.4$

This is a Square Adfected Æquation of the Third Form.

Let it be made $ba - aa = G$.

This Form used to be Resolved, by compleating the Square (as before) but first the Æquation is supposed to be changed, and made

$aa - ba = - G.$



Then $aa - ba + \frac{1}{4}bb = \frac{1}{4}bb - G$.

And $a - \frac{1}{2}b = \sqrt{\frac{1}{4}bb - G}$. Hence, $a = \frac{1}{2}b \pm \sqrt{\frac{1}{4}bb - G}$.

But I shall solve this by my Method,

Thus,	1	$r + e = a$
$1 \circledcirc 2$	2	$rr + 2re + ee = aa$
$1 \times b$	3	$br + be = ba$
$3 - 2$	4	$br - rr + be - 2re - ee = ba - aa = G$
$4 \div 2$	5	$\frac{1}{2}br - \frac{1}{2}rr + \frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G$

Then will $\frac{1}{2}be - re - \frac{1}{2}ee = \frac{1}{2}G + \frac{1}{2}rr - \frac{1}{2}br = D$.

From

From hence ariseth this

Theorem $\left\{ \dfrac{D}{\frac{1}{2}b - r - \frac{1}{2}e} = e. \right.$

$\dot{4}2,\dot{9}3 = b$ $\qquad$ $\dot{4}1\dot{8},\dot{8}4\dot{4}4 = G.$

If $r = 20$, then $rr = 400$, and $br = 858,6$

But $858,6 - 400 = 458,6$ *Ergo*, $r < 20$.

Let $r = 10$

$+ \frac{1}{2}rr = 50$ $\qquad$ $418,8444 = G$

$- \frac{1}{2}br = 214,65$ $\qquad$ $209,4222 = \frac{1}{2}G$

$- 164,65 =$ $\qquad$ $164,65 = \frac{1}{2}rr - \frac{1}{2}br$

$\frac{1}{2}b - r = 11,465)$ $\qquad$ $44.7722 = D$ $\qquad$ $(4 = e$

$- \frac{1}{2}e = 2$ $\qquad$ $37,860$

Divisor $9,465$ $\qquad$ $6,9122$ $\qquad$ $(,9 = e$

$6,3135$

New $r = 14,$

$\frac{1}{2}b - r = 7,465$ $\qquad$ 59870 $\qquad$ $(9 = e$

$- \frac{1}{2}e = ,45$ $\qquad$ 58680

Divisor $7,015$ $\qquad$ 119000 $\qquad$ $(1 = e$

64745

New $r = 14,9$ $\qquad$ • 542550 $\qquad$ $(838 = e$

$\frac{1}{2}b - r = 6,565$ $\qquad$ 517960

$- \frac{1}{2}e = ,045$ $\qquad$ 24590

Divisor $6,520$ $\qquad$ 19422

New $r = 14,99$ $\qquad$ 5168

$\frac{1}{2}b - r = 6,475$ $\qquad$ 5176

$- \frac{1}{2}e = ,0005$ $\qquad$ &c.

Divisor $6,4745$

• Here I desist forming a new Divisor.

First $r = 10,$

$+ e \ 4,991838$

$a = 14,991838$ &c.

This Third Form is something more troublesome than they Two first; but a little practice will render it very easie. And this is also a continued Series, (as before.)

PROB. IV.

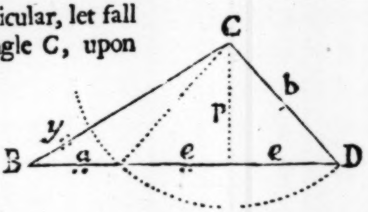
IN the Oblique Triangle BCD, there is given the Base (or Side) BD, with the Side CD; and if the Difference of the Sides BC, and CD, to wit, $BC - CD$, be Multiplied into the Side CD, that Product must be equal to the Square of the Segment of the Base, *viz.* $= aa$.

By these to find out the Segment, a.

Let	1	$BD = d$ } given { $d = 92$
	2	$CD = b$ } given { $b = 56$
And	3	$BC - DC = y$
Then	4	$by = aa$, by the Problem.

Suppose a Perpendicular, let fall from the Obtuse Angle C, upon the Side BD, call it p.

Also let $\frac{d-a}{2} = e$, as in the Scheme.



Then

Prop.	5	$pp + ee = bb$
	6	$aa + 2ae + ee + pp = bb + 2by + yy$
6 — 5	7	$aa + 2ae = 2by + yy.$
That is,	8	$a + 2e : 2b + y :: y : a.$
		And this proves Proposition, 35, 36, 37. *Euc.* 3.
To wit,		That $BD : BC + CD :: BC - CD : a.$
8, ∵	9	$2by + yy = da$, for $d = a + 2e.$
4 × 2	10	$2by = 2aa$
4 ♋ 2	11	$bbyy = aaaa$
11 ÷ bb	12	$yy = \frac{aaaa}{bb}$
10 + 12	13	$2by + yy = 2aa + \frac{aaaa}{bb} = da$
13 × bb	14	$2bbaa + aaaa = bbda$
14 ÷ a	15	$aaa + 2bba = bbd.$

In Numbers, $aaa + 6272a = 288512 = G$.

Let it be made $aaa + ba = G$.

Put	1	$r + e = a$
1 ☉ 3	2	$rrr + 3rre + 3ree = aaa$
$1 \times b$	3	$br + be = ba$
$2 + 3$	4	$r^3 + br + 3rre + be + 3ree = G$
$4 \div 3$	5	$\frac{1}{3}r^3 + \frac{1}{3}br + rre + \frac{1}{3}be + ree = \frac{1}{3}G$
$5 \div r$	6	$\frac{1}{3}rr + \frac{1}{3}b + re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r}$
6 —	7	$re + \frac{\frac{1}{3}be}{r} + ee = \frac{\frac{1}{3}G}{r} - \frac{1}{3}rr - \frac{1}{3}b = D$.

From hence ariseth this *Theorem* $\left\{ \frac{D}{r + \frac{\frac{1}{3}b}{r} + e} = e. \right.$

$6272 = b \qquad 288512 = G$

If $r = 40$, $rrr = 64000$, $br = 250880$.

But $250880 + 64000 = 314880$. *Ergo* $r < 40$.

Let $r = 30$

$\frac{1}{3}rr = 300 \qquad 26170,66 = \frac{1}{3}G$

$\frac{1}{3}b = 2090,6 \qquad 3205,688 = \frac{1}{3}G \div r$

$- 2390,6 = 2390,666 = \frac{1}{3}rr + \frac{1}{3}b$

$\frac{\frac{1}{3}b}{r} + r = 99,68 \;)\; 815,022 = D$

$+ e = 7 \qquad 742 \qquad (7 = e$

Divisor $106, \qquad 73$

New $r = 37, \qquad 2599,207207 = \frac{1}{3}G \div r$

$\frac{1}{3}b \div r = 56,504 \qquad -2546,999999 = \frac{1}{3}rr + \frac{1}{3}b$

$\frac{\frac{1}{3}b}{r} + r = 93,504 \qquad 52,207208 = D$

$47,00 \qquad (,5 = e$

$+ e = ,5 \qquad 52072$

Divisor $94,0 \qquad 47025 \qquad (5 = e$

$+ e = ,05 \qquad 504708$

Divisor $= 94,05 \qquad 470295 \qquad (5 = e$

$+ e = ,005 \qquad 344130$

Divisor $= 94,059 \qquad 282177 \qquad (3 = e$

Last $r = 37,$
$+ e = ,5553$ $\Big\} = 37,5553 = a$.

PROB.

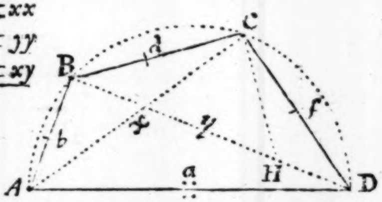
PROB. V.

THE Three Chords (or Subtences) of three Arches, compleating a Semicircle, being each given: Thence to find the Diameter of that Circle.

Suppose, $AB = 3 = b$, $BC = 4 = d$, and $DC = 5 = f$.

To find $AD = a$, the Diameter, Draw the Two Diagonals, $BD = y$, and $AC = x$. Then is

Pro. 1	1	$aa - ff = xx$
	2	$aa - bb = yy$
And	3	$da + bf = xy$



This Third Step I have often heard asserted, but never could see it clearly demonstrated, which I undertake to do in this manner.

Make CH Perpendicular to BC, then the Triangles ACD, and BCH, are alike, both Right angled at C, and having the Angles at A and B, both measured by the Arch CD. Again, the Triangles ABC, and CHD are alike; for the Angles at A, and D, stand both upon the Arch BC, and the Angles DCH, and BCA are equal; For the Angles $BCA + ACH = 90$ Degrees, also the Angles $DCH + ACH = 90$ Degrees. That is, the Angles $DCH + ACH = BCA + ACH$; and consequently the Angle $DCH = BCA$; these being proved, it will be $AC : AB :: DC : DH$, and $AC : AD :: BC : BH$.

That is,	4	$x : b :: f : DH$, and $x : a :: d : BH$
$4 \because$	5	$\frac{bf}{x} = DH$, and $\frac{da}{x} = BH$, but $y = DH + BH$
therefore	6	$\frac{bf}{x} + \frac{da}{x} = y$. Hence $bf + da = xy$.
1×2	7	$aaaa - aaff - bbaa + ffbb = xxyy$
3 ⓖ 2	8	$ddaa + 2dbfa + bbff = xxyy$
7, 8,	9	$aaaa - ffaa - bbaa = ddaa + 2dbfa$
$9 \div a$	10	$aaa - ffa - bba - dda = 2dbf$

Which in Numbers will be, $aaa - 50a = 120$.

Let

Let it be made $aaa - ba = G$. This is an Æquation of the Second Form of Cubicks, and may be solved with the same Theorem of the last, only changing the Sign of b.

Then it will be $re - \frac{\frac{1}{3}b}{r}e + ee = \frac{\frac{1}{3}G}{r} + \frac{1}{3}b - \frac{1}{3}rr = D$

From hence ariseth this

$$\textit{Theorem}\left\{\frac{D}{r - \frac{\frac{1}{3}b}{r} + e} = e\right.$$

Let $r = 6$, then $rrr = 216$, and $br = 300 > 216$ *Ergo* $r > 6$.

Suppose $r = 8$, $rrr = 512$, and $br = 400$, then $512 - 400 = 112$, hence it appears that $r > 8$, but not 9.

Let $r = 8$	$120 = G$	$b = 50$
$-\frac{1}{3}rr = 21{,}3333$	$40 = \frac{1}{3}G$	
$+\frac{1}{3}b = 16{,}6666$	$5{,}0000 = \frac{1}{3}G \div r$	
$-4{,}6667 =$	$4{,}6667 = \frac{1}{3}b - \frac{1}{3}rr$	
$\frac{1}{3}b \div r = 2{,}083$	$,3333 = D$	
$r - \frac{\frac{1}{3}b}{r} = 5{,}917$	2980	(,055 = e
	,35300	
$+e = ,055$	29860	
1 Divisor 5,96		
2 Divisor 5,972		

New $r = 8{,}955$		
$-\frac{1}{3}rr = 21{,}627675$	$40 = \frac{1}{3}G$	
$+\frac{1}{3}b = 16{,}6666666$	$4{,}9658597144 = \frac{1}{3}G \div r$	
$-4{,}9610083 =$	$4{,}9610083333$	
$r - \frac{\frac{1}{3}b}{r} = 5{,}9859$	$,0048513811 = D$	
	478936	(,0008 = e
$+e$,0008	620211	
Divisor 5,9867	598671	(1 = e
$+e$,00001	2154000	
Divisor 5,98671 *	1796013	(0359 = e
*Here I desist increasing the Divisor with e.	3579980	
	2993355	
Last $r = 8{,}055$	5866250	
$+e = ,000810359$	5389039	
$a = 8{,}055810359$		

PROB.

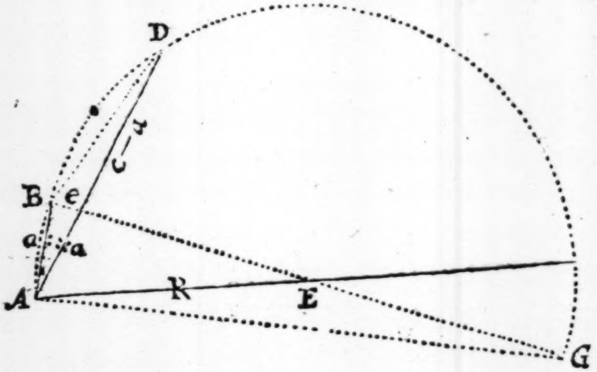
PROB. VI.

THE Radius of a Circle, and the Chord (or Subtense) of an Arch being given, thence to find out the Chord of one Third part of that Arch, which is the Trisecting of an Arch, or Angle.

Let $AE = R$, the Radius, and $AD = c$, the given Chord, (suppose of 60 Degrees.)

To find $AB = a$, (the Chord of 20 Degrees.)

Let $R = C = 1,000000$, &c.



Pro. 2. {	1	$R : a :: a : e$
	2	$c - a : e :: 2R - e : a$ $(2R - e = BG - e)$
1 ∵	3	$\frac{aa}{R} = e$
2 ∵	4	$ca - aa = 2Re - ee$
3 ⊛ 2	5	$\frac{aaaa}{RR} = ee$
$2 \times 2R$	6	$2aa = 2Re$
$6 - 5$	7	$2aa - \frac{aaaa}{RR} = 2Re - ee = ca - aa$
$7 \times RR$	8	$2RRaa - aaaa = RRca - RRaa$
$8 \div a$	9	$2RRa - aaa = RRc - RRa$
$9 +$	10	$3RRa - aaa = RRc.$

This is the Third Form of Cubick Æquations.

In Numbers, $3a - aaa = 1.$

Let

Let it be made $ba - aaa = G$.

Put	1	$r + e = a$
1 ⊛ 3	2	$r^3 + 3rre + 3ree = aaa$
1 × b	3	$br + be = ba$
3 — 2	4	$br - r^3 + be - 3rre - 3ree = G$
2 ÷ 3	5	$\frac{1}{3}br - \frac{1}{3}rrr + \frac{1}{3}be - rre - ree = \frac{1}{3}G$
5 ÷ r	6	$\frac{1}{3}b - \frac{1}{3}rr + \frac{\frac{1}{3}b}{r}e - re - ee = \frac{1}{3}G \div r$
6 ±	7	$\frac{\frac{1}{3}b}{r}e - re - ee = \frac{\frac{1}{3}G}{r} + \frac{1}{3}rr - \frac{1}{3}b = D.$

From hence ariseth this *Theorem* $\left\{ \frac{D}{\frac{\frac{1}{3}b}{r} - r - e} = e. \right.$

Being $c = 1.$ $r > ,3$ but $r < ,4.$

Let $r =$,3	$1 = G$	$b = 3$
$\frac{1}{3}rr =$,03	,333333 $= \frac{1}{3}G$	
$-\frac{1}{3}b =$ 1,00	1,111111 $= \frac{1}{3}G \div r$	
— ,97 =	,97 $= \frac{1}{3}rr - \frac{1}{3}b$	
$\frac{1}{3}b \div r =$ 3,3333	,141111 $= D$	(,04 $= e$
$-r =$,3	119732	
	213790	(7 $= e$
$\frac{\frac{1}{3}b}{r} - r =$ 3,0333	209041	
$-e$,047		
Divisor 2,9933		
Divisor 2,9863		

New $r =$,347	,9606147934 58 $= \frac{1}{3}G \div r$	
$\frac{1}{3}rr - \frac{1}{3}b = -$	,959863666667	
$\frac{\frac{1}{3}b}{r} - r =$ 2,534844)	,000751126791 $= D$	
$-e$,0002	50692	(,0002 $= e$
Divisor 2,5346	2442067	(9 $= e$
$-e$,00009	2281095	
Divisor 2,53455	16097291	(6 $= e$
$-e$,000006	15207288	
Divisor 2,534548	8900030	(35 $= e$
Last $r =$,347	7603644	
$+e$,00029635	1296386	
$a =$,34729635	1267270	

In

In the preceding Examples, I have inserted such Problems, as would produce the Three Forms (as they are usually called) of Square, and Cubick Adfected Æquations, (each in it's Order,) that is, such as consist of the highest Power thereof, mixt with its Root.

How other Æquations that are Mixt, or Adfected with their intermediate Terms, (or Inferiour Powers) to wit,

$$aaa + baa - ca = d, \text{ or } baa - ca - aaa = d, \text{\&c.}$$

may be reduced to some of the Three foregoing Forms, might here be shewed, by those commonly called *Cardan's* Rules, or otherwise; but such Process are too large and tedious, to be inserted in this small Tract: Neither (in truth) have we any need of such Expedients, as the casting off the Second Term of an Æquation, *&c.* which was invented purely to render the same more fit and easie for a Solution in Numbers. For by the Methods of Converging Series, any Æquation may be as well solved, (still keeping its own Form, as the Problem produced it,) as those of the Three Forms.

I have also omitted the Doctrin of *Surds* for the like Reason, to wit, because it requires a large Explanation to render it intelligible, without which, it's very intricate and troublesom, and rather puzles or confounds a Learners Genius, than improves it, neither could I ever (yet) perceive any great Use or Advantage thereby, in Resolving any Problem, (and afterwards we have now no occasion for it.) For admit a Problem (or any part thereof) were proposed in *Surds*, it's much easier (in my Opinion) to bring it out of such *Surds*, before they be ingaged in any Process, than after they are become intangled, and mixt with other Quantities concern'd therein.

How such Work may be performed, is already shewed.

I could likewise have inserted Rules for discovering of the several Roots of an Adfected Æquation, whether Affirmative or Negative, *&c.*

But I content my ſelf with one Root only, that is, ſuch a one as ſuits or agrees beſt with the Nature and Deſign of the Problem propoſed, Leaving thoſe Speculations to be inquired after (by ſuch as deſire them) in the Works of *Des Cartes*, or Dr. *Wallis*'s Treatiſe of *Algebra*, wherein they may meet with ample ſatisfaction in each particular caſe, *&c.*

Before I proceed to the Reſolving of other kinds of Adfected Æquations, it may be convenient to ſay ſomething of thoſe that are already done.

Perhaps it may be ſuggeſted by ſome, that this Method requires too many Operations in Diviſion, as Two, Three or ſometimes more, according to the Number of Terms in the Æquation.

That it doth require ſeveral Diviſions is true, but withal conſider, that thoſe Diviſions are performed with ſmall Diviſors, and that each Operation is in order to bring down both the Reſolvend and Diviſor into lower Terms, which was the very thing I firſt aimed at.

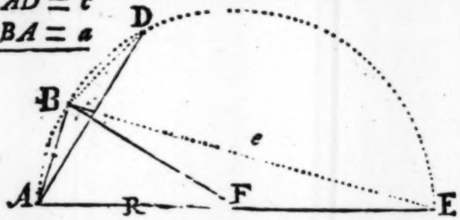
However, to accommodate ſuch as prefer *Multiplication* (though the Factors be large) before *Diviſion*, (notwithſtanding the Diviſors be ſmall,) I will here inſert a different Method of ordering the ſeveral Powers of the Root r, with the Coefficients, *&c.* which perhaps may ſeem eaſier to ſome, and therefore may pleaſe better; tho both come to the ſame in the concluſion.

And ſince I have entered upon the Subject of Angular Sections, I will proceed a little further therein; and Reſolve ſome of the odd Sections, *viz.* The Fifth, Seven and Ninth, paſſing over thoſe of the Even Powers, to wit, the Fourth, Sixth and Eight, becauſe they may be Reſolved by that of the Biſection only; for that Reaſon it will be convenient (though out of its due courſe) to inſert the Biſection of an Angle.

PROB. VII.

THE Radius of a Circle, and the Chord of an Arch, being each given, to find the Chord of half that Arch, which is to bisect an Arch, or Angle.

Let	1	$AF = R$ the Radius $= AD$, Chord of 60 Deg.
	2	$AD = c$
	3	$BA = a$



$R = 1$,
$C = 1$,
Let $BE = e$

Then

Prop. 1.	4	$4RR - aa = ee$
Prop. 2.	5	$a : c :: R : e$, therefore it is
5.	7	$aa : cc :: RR : ee$
That is	8	$aa : cc :: RR : 4RR - aa = ee$
7 ∵	8	$4RRaa - aaaa = RRcc$.

Which in Numbers is, $4aa - aaaa = 1$

Let $r + e = a$, as before,
Then will $- rrrr - 4rrre - 6rree = - aaaa$
And $+ 4rr + 8re + 4ee = + 4aa$.

By the *Prob.* $r > ,5$ but $r < ,6$ therefore
Let $r = ,5$ then $rr = ,25$ $rrr = ,125$
and $rrrr = ,0625$ then it will be

$$\left.\begin{array}{l} - ,0625 - ,5e - 1,5ee = - aaaa \\ + 1,0000 + 4,0e + 4,0ee = 4aa \end{array}\right\} = 1$$

That is, $+ ,9375 + 3,5e + 2,5ee = 1$
And $3,5re + 2,5ee = ,0625$
Then will $1,4e + ee = ,025 = D$.

Consequently

Consequently, $\dfrac{D}{1,4+e} = e$

$+e = ,0176$ added to $1,4$; Divisor $1,4176$

$,0250 = D$ (,0176 = e)
141
10900
9919
98100
85056

First $r = ,5$
$+e = ,0176$
New $r = ,5176$

I take but $,517 = r$, which being involved as before, the Numbers will be

$$- \;,0714434 - ,55275e - 1,6037ee$$
$$+ \;1,0691560 + 4,13600e + 4,0000ee$$

That is $,9977126 - 3,58425e + 2,3963ee = 1$
Then $3,58425e + 2,3963ee = ,0022874$
And $1,49593e + ee = ,0009547120 = D$

Consequently, $\dfrac{D}{1,49593+e} = e$

Divisor		Division	
	1,49593	,0009547120 = D	
+e =	,0006	89790	(,0006 = e
Divisor	1,4965	568120	
+e =	,000037	448968	(37 = e
Divisor	1,496567 *	11915200	
		10475969	
		* 14392310	(9616 = e
		13469103	
		923207	
		897936	
		25271	
		14965	
		10306	
		&c.	

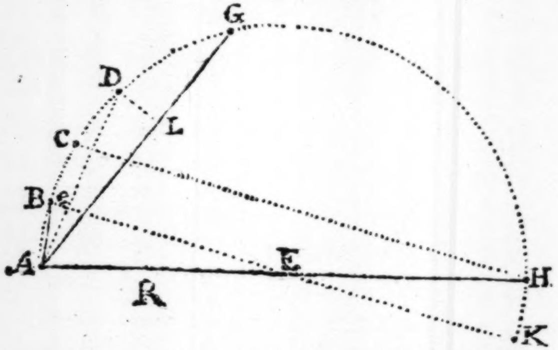
* Here I desist increasing the Divisor.

Last $r = ,517$
$+e = ,0006379616$
$a = ,5176379616$

PROB. VIII.

THE Radius of a Circle, and the Chord of an Arch (suppose of 60 gr. as before) being given, to find the Chord of one fifth part of that Arch (to wit, of 12 *gr.*)

Let $AE = R$, Radius, and $AG = c$, Chord, } given { $R = 1$, $c = 1$ }
$AB = a$, the Chord sought.



By the Third Step in the Process of the Trisection of an Angle, *Pag.* 70, it was found that $\frac{aa}{R} = e$ and by the Tenth Step, $\frac{3RRa - aaa}{RR} = AD$

Make DL, perpendicular to AD.
Then are the Triangles AHC, and DAL, alike.
And $AH : CH :: AD : AL$. But $AH = 2R$
But $CH = BK - e$ that is, $2R - \frac{aa}{R}$

Again, $AD = \frac{3RRa - aaa}{RR}$, from hence it will be

$$2R : 2R - \frac{aa}{R} :: \frac{3RRa - aaa}{RR} : AL$$

That is, $\frac{6R^4a - 5RRaaa + a^5}{2RRRR} = AL$

But

But $AL = \frac{1}{2}AG + \frac{1}{2}CD$, and $\frac{1}{2}CD = \frac{1}{2}a$

therefore, $\dfrac{6R^4a - 5RRaaa + aaaaa}{2RRRR} = \frac{1}{2}c + \frac{1}{2}a$,

That is, $aaaaa - 5RRaaa + 5RRRRa = RRRRc$

In Numbers $aaaaa - 5aaa + 5a = 1$

Put $r + e = a$

$$\text{Then}\begin{cases} +rrrrr + 5rrrre + 10rrree = aaaaa \\ -5rrr - 15rre - 15ree = -5aaa \\ +5r + 5e \ldots\ldots\ldots\ldots = 5a \end{cases}$$

If $c = 1.$ $r > ,2$ but $r < ,3$ by the Scheme,

$$\text{Then}\begin{cases} +,00032 + ,008e + ,08ee = aaaaa \\ -,04 - ,600e - 3,00ee = -5aaa \\ +1,0 + 5,0e \ldots\ldots\ldots\ldots = 5a \end{cases}$$

That is, $+,96032 + 4,408e - 2,92ee = 1$

Then $4,408e - 2,92ee = ,03968$

And $1,5e - ee = ,01356763 = D$

Consequently, $\left\{ \dfrac{D}{1,5 - e} = e \right.$

$$\begin{array}{rl} & 1,5 \\ -e = & \underline{,009} \\ \text{Divisor} & 1,491 \end{array} \qquad \begin{array}{l} ,01356763 = D. \\ \underline{13419} \\ \quad 14863 \end{array} \qquad (,009 = e$$

First $r = ,2$

$+ e = ,009$

New $r = ,209$ which being ordered as before,

$$\text{Then}\begin{cases} +,0003987782 + ,00954e + ,09129ee \\ -,0456466450 - ,65521e - 3,135000ee \\ +1,045 \qquad + 5,0e \end{cases}$$

That is, $,9997521332 + 4,35433e - 3,0437067ee$

Then $4,35433e - 3,0437ee = ,0002478668$

Each part divided by $3,0437$ then it will become

$1,430604e - ee = ,000081436015 = D$

Consequently $\left\{ \dfrac{D}{1{,}430604 - e} = e \right.$

```
        .  .  .  .  .  .  .
  1,430604)   ,000081436015 = D
— e = ,00005          715275      (,00005 = e
Divisor 1,43055       9908515     (6 = e
— e = ,000006         8583288
Divisor 1,430548      132522700   (9 = e
— e = ,0000009        128749239
Divisor 1,4305471       37734610  (263 = e
                        28610942
                         9123668
                         8583282
Last r = ,209             540386
 + e = ,0000569263        429162
   a = ,2090569263        &c.
```

I have here, (according to promise, *Page* 73.) presented you with another Method of Resolving Æquations, very different from the first; For in that the Members of the Solution (to wit, $r + e$) together with the Coefficients are so ordered, that from them (still remaining in their Species) there ariseth a Theorem for the Solution of the Æquation.

But in this Method, it is otherwise; for when the Members $r + e$, are involved according to the Æquation, and Multiplied into their respective Coefficients, then are they to be brought into Numbers, and a due Collection made thereof, according to their Signs and Places.

Next the Absolute Numbers therein (unmix'd with e) are to be compared with the Resolvend, and Added to, or Substracted therefrom as their Sign denotes; after which, each part is to be divided by the Factor found, prefixed to ee, that so the same may be cleared: Consequently, a Divisor will arise, which must be either increased or lessened by the Converging e, according to its Sign, the which, *I prsume*, doth plainly appear, in the Solution of this and the last Æquation.

I shall give an Example or Two more, that so the Young Algebrist may the better judge of the difference betwixt this and the other, and practice which he fancies.

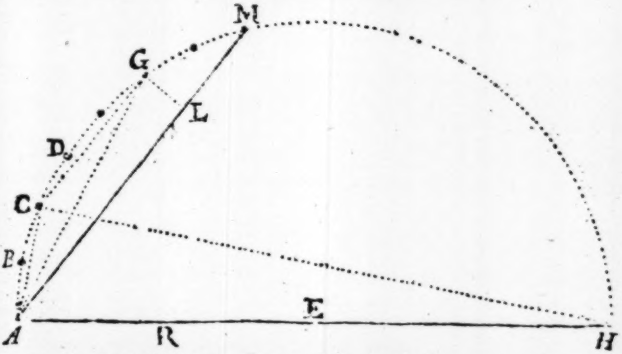
Note, That this Method doth triple the Number of Places in the Root, at each Operation, as the other doth.

PROB.

PROB. IX.

THE Radius of a Circle, and the Chord of an Arch being given, to find the Chord of the Seventh part of that Arch.

Let $AE = R$ Radius, } given { $R = 1$.
and $AM = c$ Chord, } given { $c = 1$, the Chord 0. 60.
Then $AB = a$, the Chord of 8 Degr. 34′ 19″.



By the last Problem, it was found that

$$AG = \frac{aaaaa - 5RRaaa + 5RRRRa}{RRRR}.$$

And by that in *Page* 70, it was found that

$$\frac{3RRa - aaa}{RR} = AD. \qquad \text{And } CH = \frac{2RR - aa}{R}$$

Make GL, perpendicular to AM, then are they Triangles AHC, and GAL alike; and therefore it will be

$$AH : CH :: AG : AL. \quad \text{But } AH = 2R$$

And $AL = \frac{1}{2}AM + \frac{1}{2}CG$, but $\frac{1}{2}CG = \frac{1}{2}AD$
then by taking those that are equal, it will be

$$2R : \frac{2RR - aa}{R} :: \frac{a^5 - 5RRa^3 + 5R^4a}{RRRR} : c + \frac{3RRa - aaa}{2RR}$$

If $R = 1$ as before, then it will become

$$2 : 2 - aa :: a^5 - 5aaa + 5a : \tfrac{1}{2}c + \frac{3a - aaa}{2}$$

That is, $-a^7 + 7a^5 - 14aaa + 7a = 1$

Put

Put $r + e = a$. Then it will be

$$\left.\begin{array}{l} -r^7 - 7r^6e - 21r^5ee = -a^7 \\ +7r^5 + 35r^4e + 70r^3ee = +7a^5 \\ -14r^3 - 42rre - 42ree = -14a^3 \\ +7r + 7e \ldots\ldots\ldots\ldots\ldots = +7a \end{array}\right\} = c$$

If $c = 1$ then $r > ,1$ but $r < ,2$ by the Scheme.

$$\text{Then}\left\{\begin{array}{lll} -,0000001 & -,000007e & -,00021ee \\ +,00007 & +,0035e & +,07ee \\ -,014 & -,42e & -4,2ee \\ +,7 & +7,0e & \end{array}\right.$$

That is, $+,6860699 + 6,583493e - 4,13021ee = 1$

Then $6,583493e - 4,13021ee = ,3139301$

And $1,593e - ee = ,076008 = D$

Consequently $\left\{\dfrac{D}{1,539 - e} = e\right.$

$$\begin{array}{rl} & 1,593 \\ -e = & \underline{,04} \\ \text{Divisor} & 1,55 \\ -e = & \underline{,009} \\ \text{Divisor} & 1,544 \end{array} \qquad \begin{array}{r} ,076008 = D \\ \underline{620} \\ 14008 \\ 13896 \end{array} \qquad (,049 = e$$

Last $r = ,1$ to which add $,049$ New $r = ,149$

$$\begin{array}{lll} -,00000163044 & -,0000766e & -,001542ee \\ +,00051407839 & +,0172508e & +,231566ee \\ -,046311286 & -,932442e & -6,258ee \\ +1,043 & +7,0e & \end{array}$$

$+,99720116195 + 6,0847322e - 6,027976ee = 1$

That is, $1,0094137e - ee = ,000464340511 = D$

$$\begin{array}{rl} & 1,0094137 \\ -e = & \underline{,0004602} \\ \text{Divisor} & \underline{1,0090} \\ \text{Divisor} & \underline{1,00895} \\ \text{Divisor} & 1,00895535 \end{array} \qquad \begin{array}{rl} ,000464340511 = D & \\ \underline{40360} & (,0004 = e \\ 607405 & (6 = e \\ \underline{605370} & \\ 2035 1100 & (02 = e \\ \underline{20179070} & \\ 1720700 & (01 = e \\ 1008953 & \end{array}$$

Here I desist forming a new Divisor.

$$\left.\begin{array}{l} \text{Last } r = ,149 \\ +e = ,000460201 \end{array}\right\} ,149460201 = a$$

PROB.

PROB. X.

THE Radius of a Circle, and Chord of an Arch being given, To find the Chord of one Ninth part of that Arch.

This Problem needs no Scheme, but a consideration of the Æquation arising from that in *Page* 70, *viz.* Of the Trisection of an Angle, There it was found that

$$3RRa - aaa = RRc.$$

And if it be made $R = c = 1$ in this Problem as it was in that, then it will be

$$3a - aaa = 1$$

And here we are to find a third part of that Arch whereof a, is the Chord, for $\frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ In order thereunto, let it be made $3y - yyy = c$

And for the Chord sought, put a; then will

$$3a - aaa = y, \text{ and } 9a - 3aaa = 3y$$

Also, $27aaa - 27aaaaa + 9a^7 - a^9 = yyy.$

And from these will arise this Æquation,

$$a^9 - 9a^7 + 27a^5 - 30a^3 + 9a = 3y - yyy = 0$$

If as before, $c = 1$ the Chord of 60 Degrees, then will a, be the Chord of 6 Degrees, 40 Minutes.

Put $r + e = a$, then there will arise these, *viz.*

$$+ r^9 + 9r^8e + 36r^7ee = + a^9$$
$$- 9r^7 - 63r^6e - 189r^5ee = - 9a^7$$
$$+ 27r^5 + 135r^4e + 270r^3ee = 27a^5$$
$$- 30r^3 - 90r^2e - 90ree = - 3a^3$$
$$+ 9r + 9e \ldots\ldots\ldots = 9a$$

$c = 1$ then $r >, 1$ but $r <, 2$ because we are to find the Ninth part of the Arch; Let $r = 1$ which being ordered as the Species directs, will produce these Numbers.

$+,000000001$	$+,000000009e$	$+,0000036ee$
$-,0000009$	$-,000063e$	$-,00189ee$
$+,00027$	$+,0135e$	$+,27ee$
$-,03$	$-,9e$	$-9,0ee$
$+,9$	$+9,0e$	

That

That is ,870269101 + 8,11343709*e* — 8,7318864*ee* = 1
Then 8,11343709*e* — 8,7318864*ee* = ,129730899
And ,92916*e* — *ee* = ,014857 = *D*

$$\textit{Consequently,}\ \left\{\frac{D}{,929 - e} = e\right.$$

	,92916	,014857 = *D*	
— *e* =	,0163	91	(,0163 = *e*
Divisor	,91	5757	
Divisor	,913	5478	
Divisor	,9128	27900	
		27384	

Last *r* = ,1 + ,0163 = ,1163 = *r* I take *r* = ,116
Which being ordered as before, will be

+ ,000000003829 + ,000000295*e* + ,00001017*ee*
— ,00002543598 — ,000153493*e* — ,0039642 *ee*
+ ,000567092247 + ,024443626*e* + ,4214719*ee*
— ,04682688 — 1,21104*e* — 10,44*ee*
+ 1,044 + 9,0*e*

+ ,997737671478 + 7,813250428*e* — 10,0225433*ee* = 1
That is, 7,81325*e* — 10,0225433*ee* = ,002262328522
And then, ,7795676*e* — *ee* = ,000225723995 = *D*

$$\textit{Consequently,}\ \left\{\frac{D}{,7795676 - e} = e\right.$$

	,7795676)	,000225723995 = *D*	
— *e* =	,0002896	15586	(,0002 = *e*
Divisor	,7793	698639	(8 = *e*
Divisor	,77928	623424	
Divisor	,7792780	7521595	(9 = *e*
		7013502	
The Divisor remains without new forming.		50809300	(652 = *e*
		46756680	
Last *r* =	,116	40526200	
+ *e* =	,00028965 2	38963900	
a =	,11628965 2	15623000	
		15585560	

I have chosen to prosecute these Angular Sections to this height (which is further than I ever saw) for that I was willing to shew by Example, with what ease and expedition this Method resolveth high Adfected Æquations; the Root Converging as quick in them, as it doth in those of a lower Rank.

Also because from hence may be proposed a much speedier, and easier way of approximation, towards the finding out the Circumference of a Circle, than what the Ancients had; for *Archimedes* (and others since him) proceeded therein, by the continual Bisection of an Arch, (as in *Page* 74 of this Tract) beginning with the Chord (or Subtence) of 60 Degrees; that is, of $\frac{1}{6}$ part of the Circumference, and from thence found the Chord of $\frac{1}{12}$, then of $\frac{1}{24}$, then of $\frac{1}{48}$, then of $\frac{1}{96}$, then of $\frac{1}{192}$ then of $\frac{1}{384}$, and so on, according to the nearness they intended to bring the Chord to the Arch it self. And this way of proceeding (as they resolved that Æquation) requires Twenty Seven several Extractions of the Square Root (that is, Root out of Root) to obtain the Chord of $\frac{1}{384}$, part of the Circumference. And no doubt but they made use of those Numerous Extractions, for want of other Expedients to facilitate the Work; otherwise (it is but reasonable to suppose) they would not have spent so much time and labour, as must necessarily be required in the performance thereof; both which are by this Method much abreviated, and not only so, but may be more accurately performed, because here is not required the Extraction of a Surd Root, out of a Surd Root, so often repeated; for by the Resolving of one Æquation, here is produced the Chord of $\frac{1}{54}$, part of the Circumference, beginning at the Chord of $\frac{1}{6}$, for $\frac{1}{9}$ of $\frac{1}{6} = \frac{1}{54}$; and by a Solution of the same Æquation a Second time, it will produce the Chord of $\frac{1}{486}$ part of the Circumference; for $\frac{1}{9}$ of $\frac{1}{54} = \frac{1}{486}$. And if further Exactness be required, it is but repeating it once more, and then it will produce the Chord of $\frac{1}{4374}$, part of the Circumference.

Which

Which is nearer than can be obtained by the repetition of Fifty Three several Extractions, by the Method of Bisection before mentioned.

And the better to shew how near Two Solutions will approach to the (common received) Circumference of a Circle, whose Diameter is 2, I have inserted a Second Process, taking the same Æquation equal to the Chord last found, to wit,

$$a^9 - 9a^7 + 27a^5 - 30a^3 + 9a = ,116289652$$

And for the same Reason that the first $r = ,1$ in the last Process, in this it must be made $r = ,01$

And here it is to be noted, that because $r = ,01$ then will $r^9 - 9r^7$ the Two first Members of the Solution, have one 17, and the other 13 Cyphers before their significant Figures; both which being of so small value, may be rejected; by which the Solution will become contracted to this following.

$$\left.\begin{array}{l} +27r^5 + 135r^4e + 270r^3ee \\ -30r^3 - 90r^2e - 90ree \\ +9r + 9e \end{array}\right\} = ,116289652$$

$$\text{Then} \left\{\begin{array}{lll} +,0000000027 & +,000001\,35e & +,00027ee \\ -,00003 & -,009e & -,9ee \\ +,09 & +9,000e & \end{array}\right.$$

Viz $,0899700027 + 8,99100135e - ,89973ee = ,116289652$

Then $8,99100135e - ,89973ee = ,0263196493$

And $9,993e - ee = ,0292528306 = D$

$$\textit{Consequently} \left\{ \frac{D}{9,993 - e} = e \right.$$

9,993)	,0292528306 = D	
$-e =$,002	19982	(,002928 = e
Divisor 9,991	927083	
$-e =$,0009	899109	
Divisor 9,9901	2797406	
$-e =$,00002	1998016	
Divisor 9,99008	7993900	
	7992064	

$$\left.\begin{array}{l} \text{Last } r = ,01 \\ +e = ,002928 \end{array}\right\} = ,012928 = a.$$

But

But if more Exactneſs be required, it may then be called a New r, for a Second Operation, (as before) which for brevities ſake I ſhall omit, and reſt ſatisfied with this one, as being ſufficient to ſhew how near, and with what eaſe this Approach hath been made.

This Chord ,012928 = a, is near the $\frac{1}{486}$ part of the Circumference, or Perifery of that Circle, whoſe Diameter is 2, though leſs than Juſt (it being only the ſide of the inſcrib'd Polygone) and conſequently if it be Multiplied into the Denominator of that Fraction, the Product will be near the length of the whole Perifery,

That is, ,012928 × 486 = 6,283008 the Perifery

which is ſomething leſs than that generally received, to wit, 6,283185 as may be ſeen in the works of every Geometer; ſome of which are pleaſed to call us of *Metius*, *Van Culen*, *Snellius*, or *Hugenius*, &c. as Diſcoverers of this Number. But as to the Reaſon thereof, or by what Methods it was diſcovered, therein they are ſilent, and leave the Reader to exerciſe an Implicite Faith; or be at the trouble of ſearching into the Volumes of thoſe Authors for his ſatisfaction. But herein I have ſav'd my Reader that Trouble, and (as I humbly conceive) given all, who pleaſe to conſider thereof, ample ſatisfaction, how they may with a very little labour, approach as near the Truth as can be aſſigned (for an abſolute Determination thereof cannot be obtained.) Other Methods of Approachment there are; but this by the Chord of an Arch (be it perform'd by what Section you pleaſe) I take to be the moſt Natural and Demonſtrable of all others.

Now by the help of this Perifery, 6,283008 (or if you pleaſe, that before mentioned 6,283185) and its Diameter, not only any other Perifery, but alſo the Area of any Circle may be eaſily found, I ſhall paſs over thoſe Proportions that ariſe from Squaring the Perifery, or Multiplying half the Perifery, into half the Diameter, *&c.* uſually laid down in moſt Authors, for finding the Area, And effect the ſame by a Proportion very obvious from

the Annexed Diagram. First then for obtaining the Perifery of any Circle.

Suppose the Side of the Square, $AB = 2 = D$, the Diameter of the greatest inscribed Circle, then is $6,283008 = P$ the Perifery of that Circle (as before.)

Again, Suppose the Side of the included Square, $ab = 1 = d$, the Diameter of its inscribed Circle; Let $p =$ the Perifery thereof.

Then it will be

$D : P :: d : p$

That is,

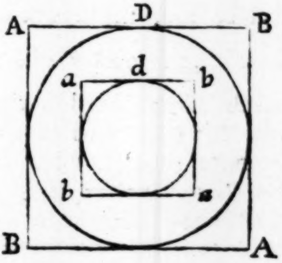
$2 : 6,283 :: 1 : 3,1415$

Consequently,

$d : p :: D : P$

That is,

$1 : 3,1415 :: D : P$



Here D is taken for any given Diameter whatsoever: Hence it follows, that if the Diameter of any Circle be Multiplied into $3,1415$ (or rather $3,1416$) the Product will be the Perifery of that Circle.

Next, for finding the Area; it's very obvious, that *As the Perimeter of the Square, Is to its contained Superficies or Area, So is the Perifery of the greatest inscribed Circle, To its Area.* (For a Demonstration of this Proportion, consider the Scheme, as if desected into parts, which will then be very obvious.)

That is, $4D : DD :: P : A =$ the Circles Area.

Or, $4d : dd :: p : a =$ its Circles Area.

In Numbers, $4 : 1 :: 3,1416 : ,7854 =$ Area.

It was before found that $d : p :: D : P$

Therefore, $4d : p :: 4D : P$ But $4d : p :: dd : a$

Consequently, $dd : a :: DD : A$

That is, $1 : ,7854 :: DD : A =$ Area.

Here

Here DD is taken for the Square of any given Diameter; from hence it follows, that if the Square of any Diameter be Multiplied into ,7854, the Product will be the Area of that Circle, in Square Measures of the same Name with the Diameter; *viz.* If the Diameter be Inches, then will the Area be square Inches, *&c.*

And what is here done, as to the Proportions betwixt the Circle and the Square, the same may be applied to the Ellipsis, and Parralellogram made of the Transverse and conjugate Diameters of that Ellipsis. But more of that (and other Conical Sections, with their Solids, *&c.*) hereafter. For if God permit, *and I find due encouragement thereunto*, I intend to present (my *Quandum* Brethren) the *Gaugers* of *Excise*, with a small Tract of *Gauging*, not only of Theorems or Rules, but also a Demonstration of each part thereof, both Algebraically, and in Words at length, with some Improvements therein not yet published. But leaving this Degression, let us proceed to some farther Application of what hath been already found.

From this Perifery 6,283008 of the Circle, whose Radius is Unity, may be easily raised a Table or Canon of Natural Sines, Tangents and Secants. In order thereto, I take it for granted, that the Sine of one Minute doth so insensibly differ from the length of the Arch of one Minute, that without Error it may be taken from the same. Then it follows, that *As the Perifery in Minutes; Is to the Perifery in Equal parts of the Radius :: So is one Minute : To the parts agreeing to that one Minute.*

That is, $21600' : 6,283008 :: 1' : ,00029088 =$ the Sine of one Minute, *which agreeth with the largest Tables I ever yet saw.* Having got the Sine of one Minute, the Co-sine thereof is thus obtained, *by the Second part of the following Scheme.*

$SD =$ Radius, $DF =$ the Sine of the Arch DA,
Then $SF = RD$ is the Co-sine of the said Arch.

 But

But the $\square SD - \square DF = \square SF.$

Therefore $\sqrt{\square SD - \square DF} = SF.$

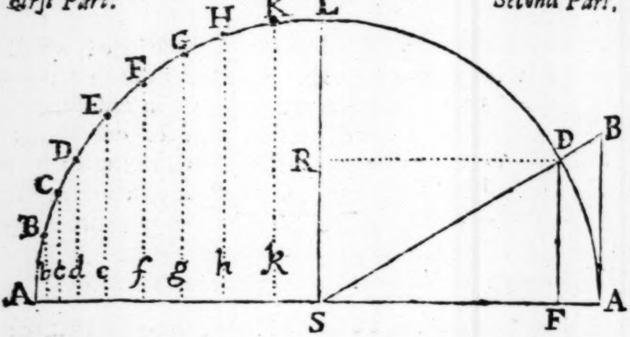
That is, From the Square of the Radius, Substract the Square of the Sine of $1'$, the Square Root of the Remainder will be the Co-sine of $1'$.

In Numbers $1 - ,0000000846\,12 = ,9999999153\,88$ The Square Root thereof is $99999995 =$ the Co-sine required.

The Sine and Co-sine of one Minute, being thus found, all the rest of the Sines in the Quadrant may be easily rais'd, by Mr. *Michael Daries's Sinical Proportion.*

Which (if I understand aright) may be thus proposed: If a Rank of Arches be equally different, such as we suppose the Arches *AB, BC, CD, DE, EF,* &c. in the Quadrant *ASL,* *of the First part of the Schem,*

First Part. *Second Part.*



whose Sines are *b, c, d, e, f, g, h, k,* and $R =$ Radius, or Sine of 90 Degrees.

Then the Proportions will run thus,

$$c : b + d :: d : c + e, \text{or } c : b + d :: g : f + h$$

Again, $f : e + g :: k : h + R$, or $d : c + e :: e : b + h$ &c.

That

That is, { *As any Sine in the Rank : Is to the Sum of the Two Sines next it on each side :: So is any other Sine in the Rank : To the Sum of the Two Sines next it on each side, or to the Sum of any Two Sines alike distant on each side it.*

From hence it follows, that if the Radius be made the Middle Term of Two Arches, then the Two Sines on each side it, may be made the Sines of Two Arches, one of them Less than a Quadrant, by one Minute, and the other of them greater than a Quadrant by one Minute; the same Sine being common to both Arches.

Then it will be $R : 2k :: b : c + o :: c : b + d$. &c.

That is, { *As the Radius : Is to the double Co-sine of one Minute :: So is the Sine of one Minute, To the Sine of Two Minutes, and of* $0'$ *:: And so is the Sine of* $2'$ *: To the Sum of the Sines of* $3'$ *and* $1'$ *:: And so is the Sine of* $3'$ *: To the Sum of the Sines of* $4'$ *and* $2'$*. And so on in a successive order.*

Now if from the Sum of the Sines of $3'$ and $1'$, be taken the Sine of $1'$, the Remainder will be the Sine of $3'$. And the like, if the Sine of $2'$, be taken from that of $4' + 2'$, the Remainder will be the Sine of $4'$, *&c.*

And by this Method of proceeding, all the Sines in the Quadrant may be easily calculated, by *Addition* and *Substraction*. For the Radius, or first Term in the Proportions being Unity, *Division* is escaped; and because the Second Term varies not, if a Tariffa, or small Table be made thereof to all the Nine Digits, then *Multiplication* is also escaped; for by the help of that Tariffa, the whole Work may be performed by *Addition* and *Substraction* only.

Having thus made the Sines, the Tangents and Secants are easily found by the following Proportions. *See the Second part of the foregoing Scheme.*

$SD = SA$ the Radius, $DF =$ the Sine of the Arch DA.

$SF =$ the Co-sine of that Arch (as before.)

$BA =$ the Tangent, and SB the Secant of the same Arch.

$$\text{Then} \begin{cases} SF : DF :: SA : BA \\ SF : SD :: SA : SB \end{cases}$$

That is, { *As the Co-sine of any Arch, is to the Sine of that Arch; So is the Radius, To the Tangent of the same Arch.* }

Again, { *As the Co-sine of any Arch, Is to the Radius; So is the Radius, To the Secant of that Arch.* }

I omit inserting the Arithmetical Operations suitable to these several Proportions, they being so obvious in themselves, there needs no Example to explain them.

Other Varieties of Proportions there are, betwixt the *Sines*, *Tangents*, *Secants*, and their *Complements*, which I forbear laying down at this time, these being sufficient to accomplish what I here design'd; which is only to shew how the Canon of Natural Sines and Tangents, *&c.* may be raised (or made) from the effects of the Tenth Problem.

Having once made the Canon of Natural Sines and Tangents, *&c.* it will not be difficult to compose the Artificial; they being only the Logarithms of the Natural; respect had to the Characteristick of the Radius, which for Conveniency (not necessity) is always made 10, as tho the Radius of the Natural were 10000000000. Notwithstanding it may be more or less at pleasure; the Characteristick of the Radius being thus assign'd, that of the other Sines will be 9, till it come to the Sine of 5 Degr. 44′, and there it changeth to 8. The Reason is plain, from the Decrease of one Place in the Natural Sines.

But

But for a further help herein, (and to gratify the Young Algebrist) I have here inserted a short Specimen of the Nature and Construction of Logarithms, which were first invented by the Lord *Nepeir*, Baron of *Merchiston* in *Scotland*; who ingeniously contrived how to perform *Multiplication* or *Division* of Natural Numbers; by Adding or Substracting certain Artificial Numbers (fitted to correspond with the Natural,) called Logarithms. This Invention of his (no doubt) arose from a mature consideration of the Mutual Agreement that is found betwixt Numbers in a Geometrical Progression, and those in Arithmetical Progression.

For Instance, Suppose a Rank of Numbers in a Geometrical Progression; and to them let there be assigned a Rank of Corresponding Numbers in Arithmetical Progression.

$$Viz.\ \begin{cases} 1\,.\,2\,.\,4\,.\,8\,.\,16\,.\,32\,.\,64\,.\,128\ \&c.\ \text{Geometrical} \\ 0\,.\,1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\ \&c.\ \text{Arithmetical} \end{cases}$$

It is very perceptible, That as the Numbers in the Geometrical Progression are produced by *Multiplication* or *Division*, those in the Arithmetical Progression are produced by *Addition*, or *Substraction*; as doth appear by this Example.

$$Viz.\ \left.\begin{cases} 4 \times 32 = 128 \\ 2 + 5 = 7 \end{cases}\right\} \text{or} \begin{cases} 128 \div 32 = 4\ \text{Geometr.} \\ 7 - 5 = 2\ \text{Arithmet.} \end{cases}$$

$$\text{Again,} \begin{cases} 1\,.\,10\,.\,100\,.\,1000\,.\,10000\,.\,100000\ \&c.\ \text{Geometric.} \\ 0\,.\,1\,.\,2\,.\,3\,.\,4\,.\,5\ \&c.\ \text{Arithmetic.} \end{cases}$$

The same Agreement is betwixt these latter, as was between the Two First Ranks.

$$Viz.\ \left.\begin{cases} 1000 \times 10 = 10000 \\ 3 + 1 = 4 \end{cases}\right\} \text{or} \begin{cases} 100000 \div 1000 = 100\ \text{Geom.} \\ 5 - 3 = 2\ \text{Arith.} \end{cases}$$

Either of these Examples do sufficiently shew the Reason, and very Ground of Logarithms.

And

And from the latter of theſe it was, that the Prime Logarithms, or Characteriſticks, were firſt aſſigned, as appears by this Table.

Natural Num.	*Logarithms.*
1	0,0000000
10	1,0000000
100	2,0000000
1000	3,0000000
10000	4,0000000
100000	5,0000000

Having laid this Foundation, the next work was to find out the Logarithms of the intermediate Numbers ſcituated betwixt 1 and 10, *viz.* of 2, 3, 4, 5, 6, 7, *&c.* And of thoſe betwixt 10 and 100, *viz.* of 11, 12, 13, 14, 15, *&c.* and ſo on for the reſt. This was a Work of ſome Difficulty, and very Laborious.

The Firſt Step in order thereunto (as I conceive) was to find out a Rank of continual Means betwixt 10 and 1, ſo as that the laſt (and leaſt thereof) might be a mix'd Number leſs than 2, and ſo near 1, as to have ſuch a Number of Cyphers before the Significant Figures thereof, as was inteended the places of Logarithms in the Table ſhould conſiſt of. Which Means are to be found, by Extracting the Square Root of 10 (having firſt annexed a Competent number of Cyphers thereunto.) Then Extracting the Root of that Root, and ſo by a continued Extraction of Root out of Root, until there be a Root ſo qualified as before mentioned. Which to make a Table to Seven Places in the Logarithms, will require Twenty Five ſeveral Extractions, the laſt of which will produce this Number, 1,00000006862238

The next Step, was to find out a Number betwixt (1) and (0) in Arithmetical Progreſſion, that might truly correſpond with the Mean before found (betwixt 10 and 1,) ſuch a Number muſt conſequently be its Logarithm. And this

this may be found by a continual bisecting (or halving) of 1, so often as was the Number of the foregoing Extractions (to wit, Twenty Five,) the last of which Bisections will produce 0,000000029802322 &c. the true Logarithm of 1,00000006862238.

For as 1,00000006862238 by Twenty Five continued Involutions (*viz.* first into it self, then that Product into it self, and so on successively) will produce 10 so will 0,00000002980232 by the like Number of doublings and redoublings, produce 1.

This Mean (or Number) and its Logarithm being thus found, it will follow by Proportion.

As the Significant Figures of this Mean : Is to the Significant Figures of its Logarithm :: So is the Significant Figures of any Mean, betwixt any given Number and 1 : (having Seven Cyphers before such Figures : as this hath) *To the significant Figures of its Logarithm.* To which must be prefixed Seven Cyphers to compleat it. After which being doubled, and redoubled according to the number of Extractions required to produce it's corresponding Mean, will at last discover the true Logarithm of the given Number. For the clearing of this, take an Example.

Suppose it were required to find the Logarithm of the Number 2, to Seven Places. First by a continued Extraction of Root out of Root, beginning at 2, find such a Mean (or Root as before) betwixt 2 and 1, as will have Seven Cyphers before it's Significant Figures; which after Twenty Three several Extractions, will be this Number 1,00000008262958. Then according to the foregoing Proportions, it will be

6862238 : 2980232 :: 8262958 : 3588557

to which prefix Seven Cyphers, as before directed, then will 1,00000008269958 have for its Logarithm ,00000003588557 which being doubled and redoubled (as abovesaid) will produce 0,30102997958658 the true Logarithm of 2, which being contracted to Seven Places, according to the first Design (and agreeable to the

Seven

Seven Places of Cyphers,) then it will become 0,3010299 But in all the Tables that I have ſeen, the Logarithm of 2 is 0,3010300 I conceive the Reaſon is, becauſe the remaining Figures 7958658 come ſo near Unity of the laſt place in the retained Figures.

And by the ſame Method that this Logarithm of 2 is made, may the Logarithm of any other Number be found. But when once the Logarithms of a few of the Prime Numbers, *viz.* of 3 . 7 . 11 . 13 . *&c.* (that is, of ſuch Numbers as cannot be produced by the Multiplying of Two Integer Factors) are obtained, the reſt may be eaſily composed by *Addition* and *Subſtraction* only.

For as $3 \times 2 = 6$ So Log. of 3 + Log. of 2 = Log. of 6

And as $10 \div 2 = 5$ So Log. of 10 — Log. of 2 = Log. of 5

The like of all Numbers that have Aliquot Parts (that is, ſuch Integer Numbers as may be divided by Integers.) And indeed the Logarithms of ſeveral of the Prime Numbers, may alſo be obtained by *Addition* or *Subſtraction*, as might eaſily be ſhewed, and is not difficult to conceive by any one, who but duly conſiders the Nature and Deſign of Logarithms, and hath been but a little converſant in their uſe, of which I ſhall forbear ſaying any thing at this time, and keep to my firſt Deſign herein, which was to give a brief Account of the Ingenious Author's Method (as I conceive it) of making the ſame (who undoubtedly found it a very difficult Work, by reaſon there is required ſo many ſeveral Extractions of Roots out of Roots, which muſt needs render it both Troubleſome and Laborious.) Then to propoſe a different Method of raiſing the Logarithms of ſuch Prime Numbers before mentioned (which require the Extraction of Roots to obtain their reſpective Means) with one Tenth part of the Trouble and Time required by the foregoing Method. And not only ſo, but more exact; for by our preſent Methods of Converging Series, the Root of any Power how high ſoever it be, is eaſily found at one ſingle Extraction; and thereby the Errors which would ariſe by Extracting

Extracting a *Surd Root* out of a *Surd Root* (especially when often repeated) are avoided; and consequently such a Mean as may be required betwixt any Number, and Unity is thereby more exactly found.

Now how this may be performed, I here intend to shew, as briefly as I can. In order thereunto, take this as a Model.

Let $a =$ the Root; or Mean required betwixt any Number and Unity,

$$\text{Then}\begin{cases} a^2 = \Box\, a \quad . \quad a^4 = \Box\, a^2 \quad . \quad a^8 = \Box\, a^4 \\ a^{16} = \Box\, a^8 \quad . \quad a^{32} = \Box\, a^{16} \quad . \quad a^{64} = \Box\, a^{32} \\ a^{128} = \Box\, a^{64} \quad . \quad a^{256} = \Box\, a^{128} \quad . \quad a^{512} = \Box\, a^{256} \end{cases}$$

And so on successively with the Indices in Geometrical Progression, until the power of (a) be made equal to such a Term in that Progression, as that the Root, or value of (a) may have betwixt Unity, and its significant Figures, so many Cyphers, as are the intended Number of Places in the Logarithms.

For instance, let it be required to find the Mean between 10 and 1 (as in *Page* 92) then the Power of (a) must be $a^{33554432} = 10$ this Index 33554432 being the 25th Term in Geometrical Progression, which may be thus determined.

Let 1 the Characteristick, or Logarithm of 10, be divided by such a Term in Geometrical Progression, as will cause such a Number of Cyphers to be before the Significant Figures in the Quotient, as are required to be before the Figures of the Root (a) suppose 7 (as before)

Then $1 \div 33554432 = ,0000000298023 2$ &c.

which is the true Arithmetical Mean (as before found, by a continual Bisecting of 1) correspondent to that signified by (a) And therefore the value of (a) found by extracting the respective Root of $10 = a^{33554432}$ will be the Mean required,

Viz. 1,0000000686223 8 whose Log. is ,0000000298023 2

These being found, are the Foundation of the rest, as before.

Then

Then suppose it be required to find the Logarithm of any of the Prime Numbers; if you please that of 2. In order thereunto, Let a = the Root, or Mean sought betwixt 2 and 1 (as before) then must (a) be continually Involved, (as by the above Model) until its Index be equal to the greatest Term in Geometrical Progression, whose Number of Places of Figures are to be equal to the Nnmber of required Cyphers before (a) to wit 7. According to which the Power of (a) will be $a^{8388608} = 2$ (this 8388608 being the 23d Term in Geometrical Progression) consequently the respective Root of $2 = a^{\underline{8388608}}$ will be the Mean required.

Example.

Let $r + e = a$

Then will $r^{8388608} + 8388608\, r^{8388607} e$ $+ 35184367894528\, r^{8388606} ee = a^{\underline{8388608}} = 2$

Suppose $r = 1$

Then $1 + 8388608e + 35184367894528ee = 2$

That is $8388608e + 35184367894528ee = 1$

Each part being divided by the Coefficient found prefixed to ee, *viz.* 351843 &c. Then it will become

$$,00000023e + ee = ,0000000000000284 = D$$

Consequently, $\left\{ \dfrac{D}{,00000023 + e} = e \right.$

$$\begin{array}{lll} & & ,0000000000000284 = D \\ & ,00000023 & \underline{248}\ (,00000008 = e \\ +e = & \underline{,00000008} & 36 \\ \text{Divisor} & ,00000031 & \end{array}$$

$$\begin{array}{rl} \text{First } r = & 1, \\ + e = & \underline{,00000008} \\ \text{New } r = & 1,00000008 \end{array}$$

Which being duly Involved (in the same order as the Model denotes) and Multiplied into the respective Coefficients, will

will then produce these Numbers

Viz. 1,9563638967 + 16411168*e* + 68833416066289*ee* = 2
Then 16411168*e* + 68833416066289*ee* = ,0436361033
And ,0000002384*e* + *ee* = ,000000000000000063393 = *D*

Consequently, $\left\{\frac{D}{,0000002384+e}\right. = e$

,000000000000000063393 = *D*

	,0000002384	480 (,0000000000263 = *e*
+ *e* =	,000000000026	15393
Divisor	,000000240	14460
Divisor	,0000002410	9330
		7230

Last *r* = 1,00000008
+ *e* = ,000000000263
New *r* = 1,000000008263

I take only 1,0000000286 = *r*. The which being Involved, and ordered as before, will produce these following Numbers, *viz.*

1,999503684867 + 167773028*e* + 7035126745408 4*ee* = 2
Then 167773028*e* + 70351267454084*ee* = ,000496315133
,0000002384186*e* + *ee* = 00000000000000000705481443 = *D*

Consequently, $\left\{\frac{D}{,0000002384186+e}\right. = e$

,00000000000000000705481443 = *D*

	,0000002384186	47686 (,00000000000295 = *e*
+ *e* =	,00000000000295	2286214
Divisor	,00000023843	2146023
Divisor	,000000238447	14019143
Divisor	,0000002384481 *	11922405
		* 20967380 (879 = *e*
		19075848
		1891532
		1669136
		222396
		214596

* Here I desist forming a new Divisor, and make use of the Abridgment, as in *Pag.* 52, and elsewhere.

Last *r* = 1,0000000826
+ *e* = ,000000000029 5879
a = 1,0000000826295879

K

This

This value of $a = 1,0000000826295879$ is the Geometrical Mean betwixt 2 and 1, as was required; (agreeable to that before found, by Twenty Three several Extractions.) And by this Method of proceeding, may be found the Mean betwixt 10 and 1 *viz.* 1,0000006862238 or betwixt any other of the (before mentioned) Prime Numbers and Unity, as might easily be shewed. But for brevities sake, I shall omit giving more Examples thereof, this one being sufficient (especially to the Ingenious) if well considered, and but once understood, to shew the nature of, and manner how, to proceed upon the like occasion, of finding any proposed Mean. The next thing will be to find the Logarithm of the Nnmber, from whence such Mean was produc'd, which may be thus perform'd.

First, find its corresponding Arithmetical Mean, or Logarithm, by Proportion (as in *Page* 93,) Then Multiply that corresponding Mean (so found) into the Index Number of such Power as the Geometrical Mean was produced from; that Product will be the Logarithm of the given Number (without a continued doubling and redoubling as before.) For the clearing of this, let it be required to compleat the Logarithm of 2.

Having first found 1,0000000686 2238 the proper Geometrical Mean betwixt 10 and 1 Also its corresponding Logarithm ,0000000298 0232 (as before directed) with them, and the Mean betwixt 2 and 1, last found, *viz.* 1,0000000826295879 make use of the abovementioned Proportion (as in *Page* 93) *viz.*

$$6862238 : 2980232 :: 826295879 : 358855729$$

to which prefix Seven Cyphers to compleat it (as before) Then it will become ,0000000358855729 This Number being Multiplied into the power of a (what that is, see *Page* 96) will produce the Logarithm of 2.

Viz. $,0000000358855729 \times 8388608 = 0,30103000391352$

But according to the first Design, it is required to have but Seven Places, *viz.* 0,3010300 which is the true Logarithm of 2 without any defect.

Thus

Thus I have presented you with a New and Expeditious Method of making Logarithms, which if they were requir'd to 14 or 15 Places (I can Modestly say) they might then be made with one Twentieth part of the Time and Trouble required by the first Method.

Before I conclude, it will not be amiss to insert an Example or Two of such Æquations as have all their Terms in them. Suppose this Æquation were given, $aaa + baa - da = G$, what's the value of a?

In Numbers $aaa + 438aa - 7825a = 98508430$

$b = 438 \quad d = 7825 \quad G = 98508430$

Let $r + e = a$.

$$\text{Then} \left\{ \begin{array}{lcl} rrr + 3rre + 3ree & = & aaa \\ brr + 2bre + bee & = & baa \\ -dr - de & = & -da \end{array} \right.$$

Suppose $r = 4$ then $rr = 16$ $rrr = 6$ $brr = 64$ and $dr = 0$ (for there is no Figure under the First Point at d) Then doth $rrr + brr = 128 > 98$ *Ergo* $r < 4$

Let $r = 300$ then it will be

$$\left. \begin{array}{l} +27000000 + 270000e + 900ee \\ +39420000 + 262800e + 438ee \\ -\ \ 2347500 - \qquad 7825e \end{array} \right\} = G$$

That is, $64072500 + 524975e + 1338ee = G$

Then $524975e + 1338ee = 34435930$

And $392e + ee = 25736 = D$.

Consequently, $\left\{ \dfrac{D}{392 + e} = e \right.$

$$\begin{array}{rr} & 392 \\ +e = & 5 \\ \hline \text{Divisor} & 44 \\ +e = & 8 \\ \hline \text{Divisor} & 450 \end{array} \qquad \begin{array}{r} 25736 = D \\ 220 \\ \hline 3736 \\ 3600 \end{array} \qquad \begin{array}{l} (5 = e \\ (8 = e \end{array}$$

$$\begin{array}{rr} \text{Last } r = & 300 \\ +e = & 58 \\ \hline \text{New } r = & 358 \end{array} \quad \text{I take but } 350 = r$$

This New r being Involved and Multiplied into the

Coefficients,

Coefficients, as the Species above directs, it will be

$$\begin{array}{lll} +42875000 & +367500e & +1050ee \\ +53655000 & +306600e & +438ee \\ -\ \ 2738750 & -\ \ \ 7825e & \end{array}$$

That is, $93791250 + 666275e + 1488ee = G$
Then $666275e + 1488ee = 4717180$
And $447,76e + ee = 3170,1478 = D$.

Consequently, $\left\{ \dfrac{D}{447,76 + e} = e \right.$

$$\begin{array}{lll} & 447,76 & 3170,1478 = D \\ +e = & \underline{\ \ \ 6,99} & \underline{2718\ \ \ } \quad (6,994 = e \\ \text{Divisor} & \underline{453,\ \ } & \ \ 452,14 \\ \text{Divisor} & \underline{454,6\ \ } & \ \ \underline{409,14\ \ } \\ \text{Divisor} & 454,75 & \ \ \ \ 430078 \\ & & \ \ \ \ \underline{409275\ \ } \\ & & \ \ \ \ \ 208030 \end{array}$$

$$\begin{array}{rl} \text{Last } r = & 350, \\ +e = & \underline{\ \ \ 6,994} \\ a = & 356,994 \end{array}$$

In this Process, I humbly conceive, there is no difficulty appears; that which seemeth like one, is the placing of the first value of $e = 5$. But if it be considered, that there must be Three Places of Integer Figures in the Root (as appears by the Points) and that this 5 is to be the Second Figure thereof, it will be easie to determine that its place must be under the Second Figure of the Divisor, *viz.* 9.

This Consideration will be very helpful in all Cases where the Root consisteth of most Integers: As for Decimal Figures they place themselves.

PROB.

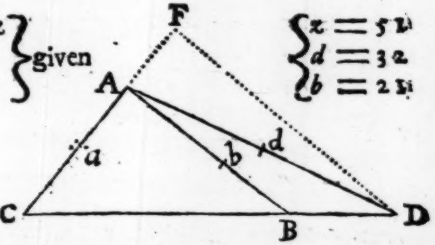
PROB. XII.

IN the Oblique Triangle, *CAD*, there is given the Side *AD*, and the Sum of the other Two Sides, *viz.* $CA + CD$; also within the Triangle, there is given the Line *AB*, so drawn, as to make the included Triangle *CAB*, Right-angled at *A*.

Thence to find out the Side *CA*.

Let

$$\left.\begin{array}{l} CA + CD = z \\ AD = d \\ AB = b \end{array}\right\} \text{given} \qquad \left\{\begin{array}{l} z = 52 \\ d = 32 \\ b = 21 \end{array}\right.$$



Draw the Line *DF*, pararallel to *AB*, and continue the Side *CA* to *F*, then they Triangles, *CAB*, and *CFD* will be alike.

Let $CA = a$ $AF = e$ and $FD = y$

Then,	1	$z - a = CD$, by the Prob.
1 ⓖ 2	2	$zz - 2za + aa = \square CD$.
but	3	$zz - 2za + aa = aa + 2ae + ee + yy$
3 − aa	4	$zz - 2za = 2ae + ee + yy$.
but	5	$dd = ee + yy$
4 − 5	6	$zz - 2za - dd = 2ae$
Let	7	$2f = zz - dd$
6, 7,	8	$2f - 2za = 2ae$
8 ÷ 2	9	$f - za = ae$
	10	$aa + bb = \square CB$, by the Scheme,
10 ω	11	$\sqrt{aa + bb} = CB$.

But	12	$CB:CA::CD:CF.$
That is,	13	$\sqrt{aa+bb}:a::z-a:a+e$
13 ∵	14	$za-aa\div\sqrt{aa+bb}=a+e$
$9\div a$	15	$f-za\div a=e$
$15+a$	16	$f-za+aa\div a=a+e.$
14, 16,	17	$\dfrac{za-aa}{\sqrt{aa+bb}}=\dfrac{f-za+aa}{a}$
$17\times a$	18	$\dfrac{zaa-aaa}{\sqrt{aa+bb}}=f-za+aa$
$18\times$	19	$za^2-a^3=f-za+a^2\times\sqrt{aa+bb}$
Then	20	$\dfrac{za^2-a^3}{f-za+aa}=\sqrt{aa+bb}$
And	21	$\square\dfrac{zaa-aaa}{f-za+aa}=aa+bb$

This 21ſt Step, or laſt Equation, being firſt Involved, then brought out of the Fraction, and reduced into Numbers, will produce this Æquation.

$$-2018aaaa+125609aaa-2464230{,}25aa$$
$$+35468307a=274183922{,}25$$

Each Member thereof being divided by 2018 the Factor of the higheſt Power of a, that ſo the ſame may be cleared, will bring down the Æquation to this,

$$-aaaa+62{,}1aaa-1221{,}12aa+$$
$$+17575{,}969a=135869{,}1388$$

Let the Æquation be made

$$-aaaa+baaa-daa+fa=G.$$

Here $62{,}1=b$, $1221{,}12=d$
and $17575{,}969=f$. $135869{,}1388=G.$

Put

Put $r + e = a$, then

$$\begin{array}{llll} -rrrr & -4rrre & -6rree & = -a^4 \\ +brrr & +3brre & +3bree & = +ba^3 \\ -drr & -2dre & -dee & = -daa \\ +fr & +fe \ldots\ldots\ldots & & = +fa \end{array}$$

Let $r = 10$, then it will be

$$\left.\begin{array}{lll} -10000 & -4000e & -600ee \\ +62100 & +18630e & +1863ee \\ -122112 & -24422e & -1221ee \\ +175750 & +17575e \ldots\ldots & \end{array}\right\} = G.$$

That is, $105547 + 7783e + 42ee = 135869$.
Then $7783e + 42ee = 30322$
And $185{,}6e + ee = 721{,}95 = D$.

$$\textit{Consequently}, \left\{ \frac{D}{185{,}6 + e} = e \right.$$

	185,6	721,95 = D	
	+ e = 3,8	564	(3,8 = e
Divisor	188	15795	
Divisor	189,4	15152	

First $r = 10$,
$+ e = 3{,}8$
New $r = 13{,}8$ I take but Two Places, to wit, $r = 13$, which being Involved, and duly Multiplied into the respective Coefficients, will produce these following Numbers.

$$\text{Viz}\left\{\begin{array}{lll} -28561{,}00 & -8788{,}00e & -1014{,}00ee \\ +136433{,}70 & +31584{,}70e & +2421{,}90ee \\ -206369{,}28 & -31749{,}12e & -1221{,}12ee \\ +228487{,}60 & +17375{,}97e & \end{array}\right.$$

That is, $129991{,}02 + 8623{,}55e + 186{,}78ee = G$
Then $8623{,}55e + 186{,}78ee = 5878{,}1188$
And $46{,}169e + ee = 31{,}470814 = D$.

Con-

Consequently, $\left\{ \frac{D}{46{,}169 + e} = e. \right.$

	46,196	31,470814 = D	
+ e =	,673	28,02	(,6736 = e
Divisor	46,7	3,4508	
Divisor	46,83	3,2781	
Divisor	46,842	172714	
		140526	
		321880	
		281052	

Last $r = 13,$
$+ e = \quad ,6736$
$a = 13,6736$

This Problem I invented, and Resolved about Sixteen Years ago (living then at *Chester*) from whence it was sent (by a Friend of his) to one *John Ward*, then and still a Mathematical Professor in *London*, from whom I was (promised) to receive a Solution thereof, but never did, either from him, or others, until I met with it in Mr *Raphson*'s before mentioned, *Analysis*, Page 24. where the same is brought to an Æquation by a different Process, to what I then did, and have here done.

This I thought convenient to insert, not for that I suppose this Problem so very difficult (tho knotty enough) but because I have here a fit opportunity to signifie to the Reader, that I am not a New Pretender to this Excellent Art, having been conversant in It (and other parts of the Mathematicks) near Twenty Years, yet never thought it proper to appear in Publick, until I had something to present the World with, worth (as I humbly conceive) their acceptance: Whether this Tract be such, or not, I leave to the Consideration of the Ingenious.

FINIS.

A SHORT
APPENDIX
CONCERNING
Compound-Interest, and *Annuities*.

MY Design in this Appendix is only to shew how such Questions in Compound-Interest, *&c.* as have heretofore been very difficult, (if not impossible) to be performed (without the help of Logarithms,) may now be easily Resolved, by the foregoing Method of Extracting Roots, without the assistance of any Tables whatsoever.

In order thereunto, I shall first touch upon those General Heads that comprehend the whole: Then lay down such Theorems, as (if well considered, and truly applied) will Resolve all Questions therein, giving some few Examples in Numbers, especially in those Questions whose Solution depends upon the Extraction of Roots.

And herein I'le not dispute the Lawfulness or Unlawfulness of this sort of Interest; only thus far it seems (to me) rational, that if it be lawful to take Interest at all, then it's as lawful to put out the Interest-Money to use, as the Principal: If so, why may it not be joyned to the first Principal (or prime Stock) as it becomes due, thereby creating (as it were) a new Principal, which if granted, then is the growing Interest properly called Interest upon Interest, or Compund Interest, the Computation of which, is no more but to find out a true, or just Equation of Payments.

For

For instance, Suppose *A* and *B*, to represent Two Persons: Then if *A* pay to *B* any Sum of Money before the time it becomes due (or ought to be paid at) it is but reasonable that *B* should make a proportional Allowance by Discount to *A*, for such time (according to the Rate of Interest agreed upon.) But on the contrary, if *A* pay not such Sum to *B*, until after the time it became due, by the same Reason *A* should make an allowance to *B*, by Amount or Increase for the time he kept such Sum after it became due. Upon these Two Cases depends all Questions relating to Interest, either Simple or Compound.

For the Resolving of which, the best way is to state or form the Question proposed, as though the Demand lay upon one Pound only: And having found a fit Answer (according to the import of the Question) for one Pound, (to a convenient Number of Decimal Parts) Multiply the Sum or Number of Pounds, *&c.* proposed in the Question, into that Answer agreeing to one Pound, the Product arising therefrom, will be the Answer required.

For the easier expressing of the several parts given or sought, I make choice of these Letters.

P = the Principal	}	given or sought in any Question.
t = the Time	}	or Number of Years or Days, *&c.* given, or sought.
a = the Amount	}	of one Pound, for one Year, or Day, *&c.* according to any Rate proposed.
z = the Sum	}	or Amount of Principal and Interest, given or sought.

Where *Note*, That in the Cases of Compound-Interest, (t) Is the Index of the Power of (a)

Now for the raising a General Theorem, by which all Questions in Compound-Interest may be Resolved, consider these Two Proportions.

First, $1l : a :: a : aa :: aa : aaa :: aaa : aaaa$ &c. in $\div\!\!\div$

That

That is, { *As one Pound : Is to its Amount, (or one Pound with its Interest) at one Years End :: So is that Account : To the Amount of one Pound at two Years, and so on.*

From hence it is evident, That Compound-Interest is grounded upon a Rank of Geometrical Proportionals Continued; the last of which is known by the Number signified by (t) and is a^t.

Secondly, $1\,l : a^t :: P : z$ *Ergo* $Pa^t = z$

That is, { *As one Pound : Is to the Account of one Pound, for any time proposed :: So is* 10 . 100 . 100 or *any Sum proposed : To its Amount, for the same time.*

From these Two Proportions, *I presume* the General Theorem $Pa^t = z$ is sufficiently demonstrated, and may be clearly understood.

Quest I. *Suppose* 250 l. *hath been at Interest Seven Years, What doth it amount to, at* 6 per Cent. per Annum, *Compound-Interest* ?

Here is given $P = 250$ $t = 7$ and $a = 1{,}06$ For $100 : 6 :: 1 : 1{,}06 = a$ the First Year; Then it follows, that if a be Involved so often, until its Index $= t$, *viz.* $a^7 = a^t$. And then Multiplied into P it will produce z as appears by the Theorem $Pa^t = z$

But $a = 1{,}06$ @ 7 times $= 1{,}50363$

And $250 \times 1{,}50363 = 375{,}9075 = z$

That is, 375 *l.* 18 *s.* 2 *d.* is the Sum produced from 250 *l.* having been at Compound-Interest 7 Years, (as above proposed)

Quest II. *Suppose* 375 l. 18 s. 2 d. *were to be paid Seven Years hence, What is it worth in ready Money, abating* 6 per Cent. per Annum, *Compound-Interest* ?

Here is given $z = 375{,}9075$ $t = 7$ and $a = 1{,}06$ to find P.

General Theorem $Pa^t = z$, therefore $z \div a^t = P$

But

But $a = 1{,}06$ Ⓖ 7 times $= 1{,}50363$

And $375{,}9075 \div 1{,}50363 = 250 = P$.

Answer it is worth 250 *l.* Ready Money.

Quest. III. *Suppose* 250 l. *hath been at Interest, and the Amount is* 375 l. 18 *s.* 2 *d.* *at* 6 per Cent. *Compound Interest, How long hath it been foreborn.*

Here is given $P = 250$ $z = 375{,}9075$ And $a = 1{,}06$ for one Year. Thence to find $t =$ the Index Power of a.

General Theorem $Pa^t = z$. Therefore $z \div p = a^t$

Consequently if a^t be continually Divided by a, until it become $a \div a = 1$ the Number of such Divisions will be $= t$. *For such Number of Divisions discovers how oft* (a) was involved.

But $375{,}9075 \div 250 = 1{,}50363 = a^t$.

And $1{,}50363 \div 1{,}06 = 1{,}418518$

Again, $1{,}418518 \div 1{,}06 = 1{,}338225$

And so on, until it become $1{,}06 \div 1{,}06$ which will be at the Seventh Operation.

Then will $t = 7$ the Number of Years required.

Quest IV. *Suppose* 250 l. *hath been forborn Seven Years, and the Debtor is willing to give up both Principal and Interest, proffering* 375 l. 18 s. 2 d. *to be cleared, What Rate of Interest* per Cent. (*allowing Compound Interest*) *doth hereby offer to the Creditor?*

Here is given $P = 250$ $z = 375{,}9075$ and $t = 7$ Thence to find a

General Theorem $Pa^t = z$ Therefore $z \div p = a^t$

That is $a^t = a^7$ *Consequently,* $\sqrt[7]{z \div P} = a$

But $z \div P = 1{,}50363 = G$

Put $r + e = a$.

$$\text{Then}\left\{\begin{array}{l} r^7 + 7r^6e + 21r^5ee = a^7 = G \\ \frac{1}{7}r^7 + r^6e + 3r^5ee = \frac{1}{7}G \\ \frac{1}{7}rr + re + 3ee = \frac{1}{7}G \div r^5 \\ re + 3ee = \dfrac{\frac{1}{7}G}{r^5} - \frac{1}{7}rr = D \end{array}\right.$$

Hence

Hence ariseth this

Theorem $\left\{ \frac{D}{r+3e} = e \right.$

Let $r = 1$ $1,50363 = G$

$,214804 = \frac{1}{7}G \div r^5$

$-\ ,142857 = \frac{1}{7}rr$

$r = 1,$

$+\ 3e = \ ,18$ $,071947 = D$ $(,06 = e$

Divisor $1,18$ 708

First $r = 1,$ $,21480428 = \frac{1}{7}G$

$+\ e = \ ,06$ $,16051432 = \frac{1}{7}G \div r^5$

New $r = 1,06$ $-\ ,16051428 = \frac{1}{7}rr$

$,00000004$

From hence it appears, that $1,06 = a$

Then $1 : 1,06 :: 100 : 6 =$ the Rate of Intereſt required.

But if in any Queſtions, either of Intereſt or Annuities, the time given or ſought, be not terminated by whole Years (as before) but lies upon Weeks, Months, Quarters, half Years, Three Quarters, &c. the beſt Method for Reſolving ſuch Queſtions, is firſt to Reduce ſuch Broken or Fractional parts of the Year into Days, viz. $\frac{1}{52} = 7$ Days, $\frac{1}{12} = 30{,}4$ Days, $\frac{1}{4} = 91{,}25$ Days, $\frac{1}{2} = 182{,}5$ Days, $\frac{3}{4} = 273{,}75$ Days, and ſo for any odd Number of Days that falls betwixt ſuch even parts of the Year. This being done, find an Anſwer, according to the Demand of the Queſtion (and agreeing to one Pound as before) for the Number of Days propoſed. Now for the performance of this (and as a Foundation for the anſwering all ſuch Demands as lies upon the parts of a Year, be they Equal or Unequal) it will be requiſite to Reſolve this following Queſtion.

What is the Amount (or if you pleaſe the Intereſt) of one Pound for one Day; at 6 per Cent. per Annum, *Compound Intereſt?*

For the Amount ſought, put a, then (according to the laſt Line in *Page* 106) it will be,

$1 : a :: a : aa :: aa : aaa :: aaa : aaaa \div$

L   That

That is, $\left\{\right.$ *As one Pound: Is to its Amount for one Day :: So is that Amount : To the Amount for Two Days :: And so is that of Two Days : To that of Three Days : And so on to 365 Days.*

The last of which will be $a^{365} = 1{,}06$

Put $r + e = a$

$$\text{Then}\left\{\begin{array}{l} r^{365} + 365r^{364}e + 66430r^{363}ee = 1{,}06 = G \\ \frac{1}{365}r^{365} + r^{364}e + 182r^{363}ee = \frac{1}{365}G \\ \frac{1}{365}rr + re + 182ee = \frac{1}{365}G \div r^{363} \\ re + 182ee = \dfrac{\frac{1}{365}G}{r^{363}} - \frac{1}{365}rr. = D \end{array}\right.$$

$$\textit{Theorem}\left\{\frac{D}{r + 182e} = e\right.$$

Let $r = 1,$ $\quad ,00290410 = \frac{1}{365}G \div r^{363}$

$+182e = ,0182$ $\quad -,00273972 = \frac{1}{365}rr$

Divisor $1,0182$ $\quad ,00016438 = D$

10182 $\quad (,00016 = e$

New $r = 1,0001$ $\quad 625600$

$+182e = ,01092$ $\quad 606612$

Divisor $1,01102$

New $r = 1,00016$ for a Second Operation.

Then is $,0027402563 6372 = \frac{1}{365}G \div r^{363}$

And $-\ ,0027406028 0986 = \frac{1}{365}rr$

Hence it appears the Excess lieth upon $\frac{1}{365}rr$, and therefore the Difference, Or New Resolvend, will have the Sign $-$ and consequently it must be $-e$

$-\ ,00000034644614 = D$

$r = 1,00016$ $\quad 30003162(-,0000003 = e$

$-182e = 1,0000546$ $\quad 46414520(-464 = e$

Divisor $1,0001054$ $\quad 40004216$

6410304

Last $r = 1,00016$ $\quad 6000630$

$-e = 000000346\,4$ $\quad 409674$

$a = 1,0001596536$ $\quad 400040$

This Value of a, is the Amount of 1 *l.* for one Day; from

from which, if 1 *l.* be Substracted, the Remainder ,0001596536 will be the Interest of 1 *l.* for one Day. Consequently, if any proposed Sum be Multiplied into either of these, the respective Product will be the Amount, or Interest of that Sum for one Day.

And if from hence, were Calculated a Table of the several Powers of a, it would then be

$$a \,.\, a^2 \,.\, a^3 \,.\, a^4 \,.\, a^5 \,.\, a^6 \,.\, a^7 = \text{the Amounts,}$$
$$1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5 \,.\, 6 \,.\, 7 = \text{the Days,}$$

and so continued on to $a^{365} = 1{,}06$ the Amount for 365 Days.

This Table would be of good use for the speedy Resolving of all Questions relating to Compound Interest, *&c.* for any number of Days less than one Year, as might be easily shew'd; but I shall omit giving Examples thereof, what hath been already deliver'd, being sufficient (if consider'd, and once understood) to render the whole Business of Compound Interest Easie: Neither indeed can that of *Annuities*, *Pensions*, and *Leases* in *Reversion*, &c. be then difficult; their difference being only this, Compound Interest is grounded upon a Rank of Geometrical Proportionals continually increasing (as in *Page* 107) the business of *Annuities*, &c. is grounded upon a Rank of Geometrical Proportionals (but) continually decreasing, And may be thus express'd.

$p =$ the $\left\{\begin{array}{l}\text{Pension, or Annuity; and is the first or greatest}\\ \text{Term in the Progression.}\end{array}\right.$

$t =$ the $\left\{\begin{array}{l}\text{Time of Continuance (as in Interest) and is}\\ \text{the Number of all the Terms except the First.}\end{array}\right.$

$a =$ the $\left\{\begin{array}{l}\text{Rate of Interest for one Pound (as before) and}\\ \text{is the common Ratio of all the Terms.}\end{array}\right.$

$z =$ the $\left\{\begin{array}{l}\text{Sum of all the Terms, except the first, and is}\\ \text{the Price, or present worth of any Annuity,}\\ \text{or Pension, \&c.}\end{array}\right.$

These premised, the Progression will be

$$p : \frac{p}{a} :: \frac{p}{a} : \frac{p}{aa} :: \frac{p}{aa} : \frac{p}{aaa} :: \frac{p}{aaa} : \frac{p}{aaaa} \;\&c. \text{ in } \div\div$$

until

until it become $\frac{P}{a^t}$ That is, until the Index of the Power of a, be equal to the time of Continuance of the Annuity. In this Progression $\frac{P}{a^t}$ is the last Term.

Then $z - \frac{P}{a^t}$ will be the Sum of all the Antecedents.

And $z - \frac{P}{a}$ will be the Sum of all the Consequents.

Then $\left\{\right.$ *As one of the Antecedents : Is to its Consequent :: So is the Sum of the Antecedents : To the Sum of the Consequents.* (vide *Page* 25)

That is, $P : \frac{P}{a} :: z - \frac{P}{a^t} : z - \frac{P}{a}$

Ergo $zP - \frac{PP}{a} = \frac{zP}{a} - \frac{PP}{a^t \times a}$

That is, $za - p = z - \frac{P}{a^t}$ Or $za - z = p - \frac{P}{a^t}$

By this Æquation may all the Cases in Annuities or Pensions, &c. (that are bounded by Time) be Resolved, by transposing and ordering the several parts thereof, according as the Nature and Demand of the Question requires.

For instance, *Suppose the Yearly Pension or Lease, Years of Continuance, and Rate of Interest were each given, Thence to find the worth thereof in present Money.*

Here is given P, t, and a, to find z.

The Æquation $za - z = p - \frac{P}{a^t}$ therefore $z = \frac{P - \frac{P}{a^t}}{a - 1}$

The like for finding any of the other Parts, *viz.* P, t, or a. I conceive it needless to suit Arithmetical Examples thereunto; they being perform'd in all respects like unto those of Interest. I shall therefore leave them to the Exercise of the Practitioner. To whom I wish all good success.

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until it become $\frac{P}{a^t}$ That is, until the Index of the Power of a, be equal to the time of Continuance of the Annuity. In this Progression $\frac{P}{a^t}$ is the last Term.

Then $z - \frac{P}{a^t}$ will be the Sum of all the Antecedents.

And $z - \frac{P}{a}$ will be the Sum of all the Consequents.

Then $\left\{\begin{array}{l}\textit{As one of the Antecedents : Is to its Consequent ::}\\ \textit{So is the Sum of the Antecedents : To the Sum of}\\ \textit{the Consequents.}\ \text{(vide } \textit{Page 25}\text{)}\end{array}\right.$

That is, $P : \frac{P}{a} :: z - \frac{P}{a^t} : z - \frac{P}{a}$

Ergo $zP - \frac{PP}{a} = \frac{zP}{a} - \frac{PP}{a^t \times a}$

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